

Infinite limit cycle bifurcation in a delayed differential equation

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Summary. We investigate the occurrence of a bifurcation, which we term the “infinite limit cycle bifurcation”, or ILCB for short, in the nonlinear DDE (delay-differential equation) $\frac{d^2x(t)}{dt^2} + x(t - T) + x(t)^3 = 0$, where T is the delay time. When $T > 0$, no matter how small, an infinite number of limit cycles (LCs) exist, their amplitude going to infinity as T goes to zero. Perturbation methods are used to approximate the system and investigate the nature of such a bifurcation.

Introduction

In a 2017 paper by Davidow, Shayak and Rand [1], it was shown that the nonlinear DDE (delay-differential equation)

$$\frac{d^2x(t)}{dt^2} + x(t - T) + x(t)^3 = 0 \tag{1}$$

exhibits an infinite number of limit cycles when $T > 0$. Since that publication, we have found two other papers that have independently discovered this same ILCB [2],[3]. A numerical integration of (1), using dde23 in matlab, shows the first eight stable limit cycles for the case when $T = 0.3$, Figure 1. In [1] an approximation of the amplitude of the limit cycles using the harmonic balance method (HB) and Melnikov’s integral was obtained. In the harmonic balance method only one term was considered ($x = A \cos wt$) yielding the amplitude: $A = \frac{2}{\sqrt{3}} \sqrt{\frac{n^2 \pi^2}{T^2} \pm 1}$, $n = 1, 2, 3, \dots$ where the upper sign refers to n odd, and the lower sign refers to n even. In this work we expand the analytical investigations in [1] to improve the approximation of the system and gain further insight into the nature of the infinite limit cycle bifurcation.

Analysis

Performing the harmonic balance method and including three terms in the expansion ($x = A \cos wt + B \cos 3wt + C \cos 5wt$), three nonlinear algebraic equations on A , B and C are obtained. Although a closed form expression for the amplitude is unavailable, a numerical solution for A , B and C and for the amplitude of the limit cycles are obtained. Table 1 shows a comparison between the approximated amplitudes obtained by the harmonic balance method with one (A), two (A,B) and three (A,B,C) terms.

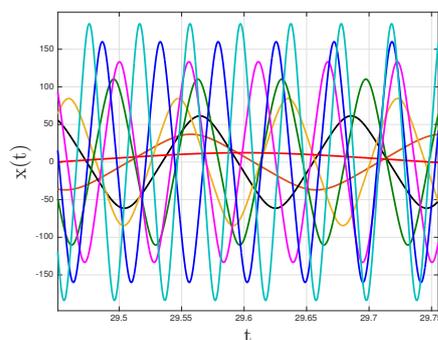


Figure 1: The first 8 stable limit cycles obtained by numerical integration of (1) for $T = 0.3$ using dde23 in matlab .

Harm. Bal. (1 term)	Harm. Bal. (2 terms)	Harm. Bal. (3 terms)
12.14	12.39	12.41
24.21	24.64	24.68
36.29	37.02	37.09
48.38	49.33	49.42
60.47	61.69	61.80

Table 1: Stable and unstable limit cycle amplitudes obtained by harmonic balance method for $T = 0.3$.

Conclusions

We have investigated the effect of including additional terms in the method of harmonic balance as applied to the ILCB. This is a work in progress; updated results will be orally presented at the conference.

References

- [1] M. Davidow, B. Shayak, R. H. Rand, Analysis of a remarkable singularity in a nonlinear DDE, *Nonlinear Dynamics*, (2017) 90:317-323.
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