Variation in Young's Modulus Along Apple Limbs

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This article is reprinted from the TRANSACTIONS of the ASAE (Vol. 30, No. 5, pp. 1501-1505, 1987)
Published by the American Society of Agricultural Engineers, St. Joseph, Michigan

ABSTRACT

A bending test on an Instron testing machine revealed a surprising result: that Young's modulus of an apple limb increases from the base of the limb to the tip of the limb. A theoretical analysis attributes this increase in apparent Young's modulus to the nonhomogeneity of the limb cross-section. The difference between Young's modulus of the bark and of the inner lignified wood, as well as effects due to the turgor in the living cells, may be responsible for the observed results.

INTRODUCTION

Most mechanical tree-fruit harvesters detach fruits by applying mechanical force to the tree trunk or tree limbs. The external force propagates through the tree trunk and/or limbs. The transmission of energy through the tree material depends upon the tree's physical properties — such as density, damping coefficient, and Young's modulus — as well as upon its geometry and upon the nature of the external force.

Upadhyaya (1979) developed a finite element model for the transient dynamics of limb impact harvesting of apple. The implementation of the finite element method required a knowledge of Young's modulus for each element. When each element of the six experimental limbs (three Red Delicious and three McIntosh) used in the 1979 study were tested for Young's modulus by means of bending tests on an Instron machine, a surprising result was found: on a given limb Young's modulus increased from the base of the limb to the tip of the limb. This trend was observed on all the limbs tested. The objectives of this study were: (a) to confirm the above observations by conducting further experiments and (b) to provide a rational theoretical model for the observed results.

REVIEW OF LITERATURE

Numerous investigations have been conducted in the past to study the physical properties of agricultural products. It was found that agricultural products such as fruits and vegetables were visco-elastic in nature (Mohsenin et al., 1963; Morrow and Mohsenin, 1966; Clevenger and Hammann, 1968 and Hammerle and Mohsenin, 1970). Therefore, both the rate and duration of loading are important (Mohsenin et al., 1963 and Murase et al., 1980). The visco-elastic behavior of wood becomes important if the loading duration is greater than 1 s (Panshin and DeZeeuw, 1970).

The elastic modulus of an agricultural material depends on the turgor of its cells (Nilsson et al., 1958; Falk et al., 1958 and Mohsenin, 1970). This overall elastic modulus depends upon, but is not equal to, the elastic moduli of the cell walls.

Murase and Merva (1977a) stressed the desirability of using cell water potential in preference to cell turgor or moisture content for predicting the effect of mechanical stress on vegetable tissue. They developed a constitutive relation for vegetative media (Murase and Merva, 1977b). DeBaerdemaeker et al., (1978) found that water potential of cells affects compressive and tensile failure stress of apple and potato tissues. Murase et al., (1980) distinguished between the apparent elastic modulus of a tissue in response to an applied strain and the true material property of cell wall material. Young's modulus of the potato tissue was found to be a linear function of the water potential at constant strain.

Diener et al. (1968) studied the directional dependence of mechanical properties of apple, peach, and cherry bark. Their results indicated that the bark was anisotropic. Diener et al. (1969) found that the elastic modulus of apple limbs shows seasonal variation. It was low in June and high in January. They attributed this increase in January to a thickening of cell sap.

Fig. 1 is a schematic of the anatomy of a stem. A stem consists of four types of tissue: (a) outer bark made up of dead epidermal and cork cells, (b) a thin layer of living cells consisting of cork cambium, cortex, phloem, cambium, and part of the xylem (young wood) — which may have high turgor, (c) the woody xylem, and (d) the central pith. The latter two portions occupy the major cross-sectional area of the stem and contribute to the commercial value of the wood. The cells in this region are nonliving and are lignified (Fuller and Tippo, 1960). A cylindrical column of wood near the pith, called juvenile wood, consists of shorter cells and has inferior physical properties. The outer adult wood is produced by mature cambium and has superior physical properties (Panshin and DeZeeuw, 1970).

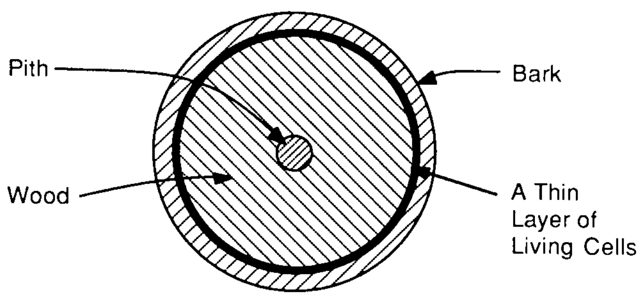

Fig. 1—A schematic of the cross-section of the stem.

Article was submitted for publication in November, 1986; reviewed and approved for publication by the Food and Process Engineering Institute of ASAE in July, 1987. Presented as ASAE Paper No. 81-3038.

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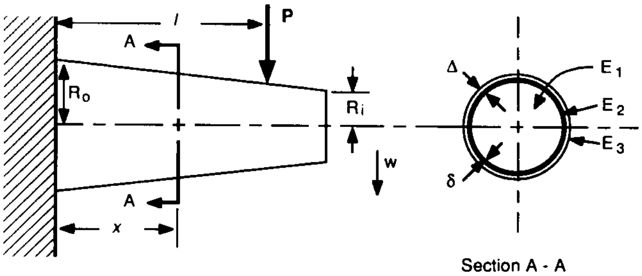

Fig. 2—A tapered composite beam supported as a cantilever beam.

MATHEMATICAL MODELING

In this section we will develop an expression for the deflection of a tapered composite beam supported as a cantilever beam (Fig. 2).

We made several assumptions to simplify the analysis. These assumptions were: (a) the deflections and the associated slopes of the neutral axis were small, (b) the deflection due to shear was negligible, (c) the plane cross-sections remained plane, (d) the stress was proportional to strain, (e) the beam was straight and tapered linearly from the base to the tip. The taper of the beam was small, (f) the cross-section of the beam was divided into three uniform layers, as shown in Fig. 1, and (g) the cross-section of the beam was assumed to be circular.

Application of Bernoulli-Euler's theory for this case gave

Strain: $\in = z/\rho$

where z = distance from the neutral axis, m

ρ = curvature (m^{-1}) and

Stress: $\sigma = E\in$

Substituting for $\in$ yielded

$\sigma = Ez/\rho$

where E = Young's modulus, Pa

Therefore, the moment,

$$M = \int_0^R \int_0^{2\pi} \frac{Ez}{\rho} z \,(rd\phi dr) \quad \ldots\ldots\ldots\ldots\ldots [1]$$

where R = beam radius, m.

But, from Fig. 2, $M = P(\ell - x)$ ....[2]

where P = the applied load, N.

ℓ = distance from the base of the beam to the point where the load is applied, m

x = distance from the base, $0 < x < \ell$ (m).

Combining equations [1] and [2] gave

$$P(\ell - x) = \int_0^R \int_0^{2\pi} \frac{Ez^2}{\rho} (rd\phi dr).$$

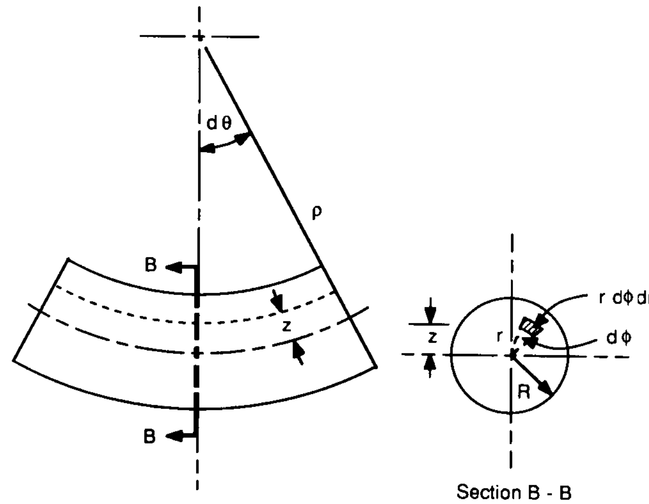

Fig. 3—Curvature of the neutral axis.

Since z = r sin ϕ (Fig. 3), substituting for z in the above equation gave

$$P(\ell - x) = \int_0^R \frac{Er^3}{\rho} \int_0^{2\pi} \sin^2\phi d\phi.$$

Integrating with respect to ϕ gave

$$P(\ell - x) = \frac{\pi}{\rho} \left[\int_0^{R_1} E_1 r^3 dr + \int_{R_1}^{R_2} E_2 r^3 dr + \int_{R_2}^{R_3} E_3 r^3 dr \right].$$

Integrating with respect to r gave

$$P(\ell - x) = \frac{\pi}{4\rho} [E_1 R_1^4 + E_2 (R_2^4 - R_1^4) + E_3 (R_3^4 - R_2^4)] \quad \ldots\ldots\ldots\ldots\ldots [3]$$

Defining

$$S = \frac{\pi}{4} [E_1 R_1^4 + E_2 (R_2^4 - R_1^4) + E_3 (R_3^4 - R_2^4)]$$

we obtained upon rearrangement

$$S = \frac{\pi}{4} [(E_1 - E_2) R_1^4 + (E_2 - E_3) R_2^4 + E_3 R_3^4].$$

If we defined $R_3 = R$, δ = thickness of the bark, Δ = thickness of the bark plus living cells, then $R_2 = R - \delta$, $R_1 = R - \Delta$, and

$$S = \frac{\pi}{4} R^4 \left\{ (E_1 - E_2)(1 - \frac{\Delta}{R})^4 + (E_2 - E_3)(1 - \frac{\delta}{R})^4 + E_3 \right\}.$$

For Δ and $\delta << R$, we obtained

$$S = \frac{\pi}{4} R^4 \left\{ (E_1 - E_2)\left(1 - \frac{4\Delta}{R}\right) + (E_2 - E_3)\left(1 - \frac{4\delta}{R}\right) + E_3 \right\}$$

which upon rearrangement of terms reduced to

$$S = \frac{\pi}{4} R^4 \left\{ E_1 - \frac{4}{R}\left[\Delta(E_1 - E_2) + \delta\,(E_2 - E_3)\right] \right\} \quad \ldots [4]$$

Because of lignification E_1 is expected to be greater than E_3. Similarly E_2 is expected to be greater than E_3 due to turgor (Nilsson et al., 1958). Therefore if we defined $E_3 = E_o$, following relationships are expected for E_1 and E_2

$$E_1 = E_o + E$$

$$E_2 = E_o + \beta P$$

where
P = turgor pressure
β = a coefficient.
Therefore,

$$S = \frac{\pi R^4}{4} \left\{ Eo + E - \frac{4}{R}\left[\Delta(E - \beta P) + \delta\beta P\right] \right\}$$

and combining terms resulted in

$$S = \frac{\pi R^4}{4}\left[Eo + E\left(1 - \frac{4\Delta}{R}\right) + \frac{4\beta P}{R}(\Delta - \delta)\right] \quad \ldots\ldots\ldots [5]$$

From equation [3]

$$\frac{1}{\rho} = \frac{P(\ell - x)}{S}.$$

If the deflection and slopes were small

$$\frac{1}{\rho} \cong \frac{d^2w}{dx^2}$$

where w was beam deflection.
Therefore,

$$\frac{d^2w}{dx^2} = \frac{P(\ell - x)}{S} \quad \ldots\ldots\ldots\ldots\ldots [6]$$

Equation [6] along with the boundary conditions

$$w = o \text{ and } \frac{dw}{dx} = o \text{ for } x = o$$

gave us the following expression for w upon integration

$$w = \frac{4\,P\ell^3}{3\pi Ro^3 Ri \left\{ E_o + E - \frac{(Ro + 3\,Ri)}{RoRi}\left[\Delta(E - \beta P) + \delta\beta P\right] \right\}} \quad \ldots\ldots\ldots\ldots\ldots [7]$$

A linear taper assumption (5) was used for R in equation [5] in the form:

$$R = Ri + \gamma x$$

where $\gamma = Ro - Ri/\ell$.
Rearranging terms in equation [7] gave

$$w = \frac{4\,P\ell^3}{3\pi Ro^3 Ri\left[E_o + E\left\{1 - \frac{\Delta(Ro+3\,Ri)}{RoRi}\right\} + \frac{\beta P(\Delta - \delta)(Ro+3\,Ri)}{RoRi}\right]}$$

$$\text{Defining } E_{av} = E_o + E\left\{1 - \frac{\Delta(Ro+3\,Ri)}{RoRi}\right\} + \beta P(\Delta - \delta)\frac{(Ro+3\,Ri)}{RoRi} \quad \ldots\ldots\ldots\ldots\ldots [8]$$

we obtained

$$w = \frac{4\,P\ell^3}{3\pi Ro^3 Ri\, E_{av}} \quad \ldots\ldots\ldots\ldots\ldots [9]$$

where
ℓ = distance from the base of the beam to the point where the load is applied, mm
w = deflection of the beam at the point where load was applied, mm
Ro = the radius of the beam at the base, mm
R_i = the radius of the beam at the tip, mm
E_{av} = average Young's modulus, Pa.

Average Young's modulus was obtained using the slope (P/w) of the force deflection curve

$$E_{av} = \left(\frac{P}{w}\right) x \left(\frac{4\,\ell^3}{3\pi Ro^3 Ri}\right) \quad \ldots\ldots\ldots\ldots\ldots [10]$$

EXPERIMENTAL METHOD

Five McIntosh limbs were tested on an Instron testing machine by loading the limbs as cantilever beams. Each limb was divided into approximately 300-mm long elements. The length to diameter ratios of all elements were maintained between 5 and 12 as required by ASTM standards (ASTM Standards, 1954). Each element was mounted on a specially prepared bracket on an Instron testing machine. The bracket was made from a four independent jaw lathe chuck and mounted on the Instron frame with angle brackets. This fixture provided the "fixed" end conditions for the cantilever beam.

A force-deflection curve was obtained for each limb segment. The measurements were repeated by holding the limb at three different locations (approximately 120 deg apart) along the circumference at the base of the element. The distance of the point of application of the load from the point of support, the average diameters of the element at the base and at the point of application of the load were measured.

The moisture content in each of the elements was measured using an oven dry method. The oven was maintained at 105 °C. The weight loss of the elements were recorded on a daily basis until no further weight loss occurred. The steady-state values were used in determining limb element moisture content.

TABLE 1. YOUNG'S MODULUS AND RELATED DATA FOR MCINTOSH LIMB ONE.

Length, (L), mm	Diameter, mm		MC*, %	Span rat.†		Young's mod., MPa
	D1	D2		R1	R2	
212.7	32.64	31.37	49	6.52	6.78	2.15E+03
215.9	31.37	30.16	49	6.88	7.16	2.03E+03
213.5	30.07	28.09	40	7.10	7.60	2.36E+03
220.7	25.65	22.16	50	8.60	9.96	4.36E+03
225.4	21.91	20.51	35	10.29	10.99	4.62E+03

*MC = moisture content
†Span rat = length/diameter for the beam element

TABLE 2. YOUNG'S MODULUS AND RELATED DATA FOR MCINTOSH LIMB TWO.

Length, (L), mm	Diameter, mm		MC*, %	Span rat.†		Young's mod., MPa
	D1	D2		R1	R2	
215.9	34.61	32.80	60	6.24	6.58	1.82E+03
219.1	31.56	30.29	60	6.94	7.23	2.40E+03
220.0	27.69	26.71	56	7.94	8.24	3.88E+03
215.1	26.35	25.21	58	8.16	8.53	3.29E+03
209.6	23.96	22.99	58	8.75	9.12	3.37E+03

*MC = moisture content
†Span rat = length/diameter for the beam element

TABLE 3. YOUNG'S MODULUS AND RELATED DATA FOR MCINTOSH LIMB THREE.

Length, (L), mm	Diameter, mm		MC*, %	Span rat.†		Young's mod., MPa
	D1	D2		R1	R2	
217.5	29.53	28.24	69	7.37	7.70	2.41E+03
209.6	27.94	28.58	60	7.50	7.33	2.59E+03
212.7	26.67	24.96	85	7.98	8.52	3.01E+03
193.7	23.94	22.92	73	8.09	8.45	3.36E+03
209.6	22.92	23.81	77	9.14	8.80	3.34E+03

*MC = moisture content
†Span rat = length/diameter for the beam element

TABLE 4. YOUNG'S MODULUS AND RELATED DATA FOR MCINTOSH LIMB FOUR.

Length, (L), mm	Diameter, mm		MC*, %	Span rat.†		Young's mod., MPa
	D1	D2		R1	R2	
215.9	25.27	24.89	73	8.54	8.67	3.34E+03
220.7	24.51	23.75	75	9.01	9.29	3.46E+03
222.3	23.05	21.17	78	9.64	10.45	4.34E+03
219.1	21.02	21.59	82	10.42	10.15	4.23E+03
212.7	19.56	19.05	80	10.88	11.17	4.43E+03

*MC = moisture content
†Span rat = length/diameter for the beam element

TABLE 5. YOUNG'S MODULUS AND RELATED DATA FOR MCINTOSH LIMB FIVE.

Length, (L), mm	Diameter, mm		MC*, %	Span rat.†		Young's mod., MPa
	D1	D2		R1	R2	
214.4	29.85	29.21	74	7.18	7.34	2.94E+03
215.9	27.75	27.50	79	7.78	7.85	3.03E+03
228.6	25.91	23.18	80	8.82	9.86	4.49E+03
220.7	23.50	22.10	79	9.39	9.99	3.96E+03
215.9	17.53	17.02	77	12.32	12.69	5.22E+03

*MC = moisture content
†Span rat = length/diameter for the beam element

TABLE 6. CORRELATION COEFFICIENTS FOR MCINTOSH LIMBS

Limb number	Correlation coefficients
McIntosh	
1	0.96
2	0.81
3	0.98
4	0.87
5	0.89

RESULTS AND DISCUSSION

The tree limb characteristics and the average Young's moduli for all the five limbs tested are shown in Tables 1 through 5. The results show an increase in Young's modulus from the base to the tip of the limb. No definite trend in the moisture content was observed along the limb. All the limbs tested had very high moisture contents. Since moisture levels were well above the fiber saturation point, they were not expected to affect the limb's strength properties (Panshin and DeZeeuw, 1970).

The observed increase in the average Young's modulus from the base of the limb to the tip was surprising. However, from equation [8]

$$E_{av} = E_o + E \left\{ 1 - \frac{\Delta(Ro+3\,Ri)}{RoRi} \right\} + \beta P(\Delta - \delta)\,\frac{(Ro+3\,Ri)}{RoRi}$$

or

$$E_{av} = E_o + E + \left(\frac{Ro+3\,Ri}{RoRi}\right)(\beta P(\Delta - \delta) - \Delta * E) \quad \ldots\ldots [11]$$

If $\beta P\ (\Delta - \delta) > \Delta^* E$, then as the radius of the limb decreased, E_{av} would increase. Therefore, under high turgor conditions one would expect the results obtained in this study. Such conditions exist during the growing season.

The correlation coefficients between Young's modulus, E_{av} and (Ro + 3 Ri)/(RoRi), listed in Table 6, were very high for all five limbs. Thus, our model suggests the observed variation in average Young's modulus was due to the turgor in the living cells of the limb. If we assume the cross-section of the limb to be homogeneous, we would expect a constant value of Young's modulus from the base of the limb to the tip. However, Dr. George Kayanka, Wood Products Engineer at SUNY CESF, Syracuse, believes that the observed variation may also be due to the increased microfibriller angle (measured from vertical) often found in wood near the pith. Our model does not account for such variations within the wood. In fruit harvesting studies, such variations may be important in the study of propagation of an impact or sinusoidal forcing function through the tree structure. Such variations can easily be incorporated into a finite element or any other numerical methods.

Before closing it should be noted that the stiffness, which was an integral of the product of Young's modulus with the area moment of inertia, S, decreased from the base of the limb to the tip because S varied as the fourth power of the diameter. Thus a slight decrease in diameter resulted in a decrease in stiffness despite the increase in Young's modulus.

CONCLUSIONS

1. We have developed an expression for the variation of Young's modulus for a three media composite tapered beam.

2. The apparent Young's modulus along an apple limb increased as location changed from the base of the limb to the tip. The observed variation was due to the non-homogeneity of the limb cross-section. The non-homogeneity was probably due to the differences in the elastic properties of the wood and bark and due to the influence of turgor in the living cells.

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