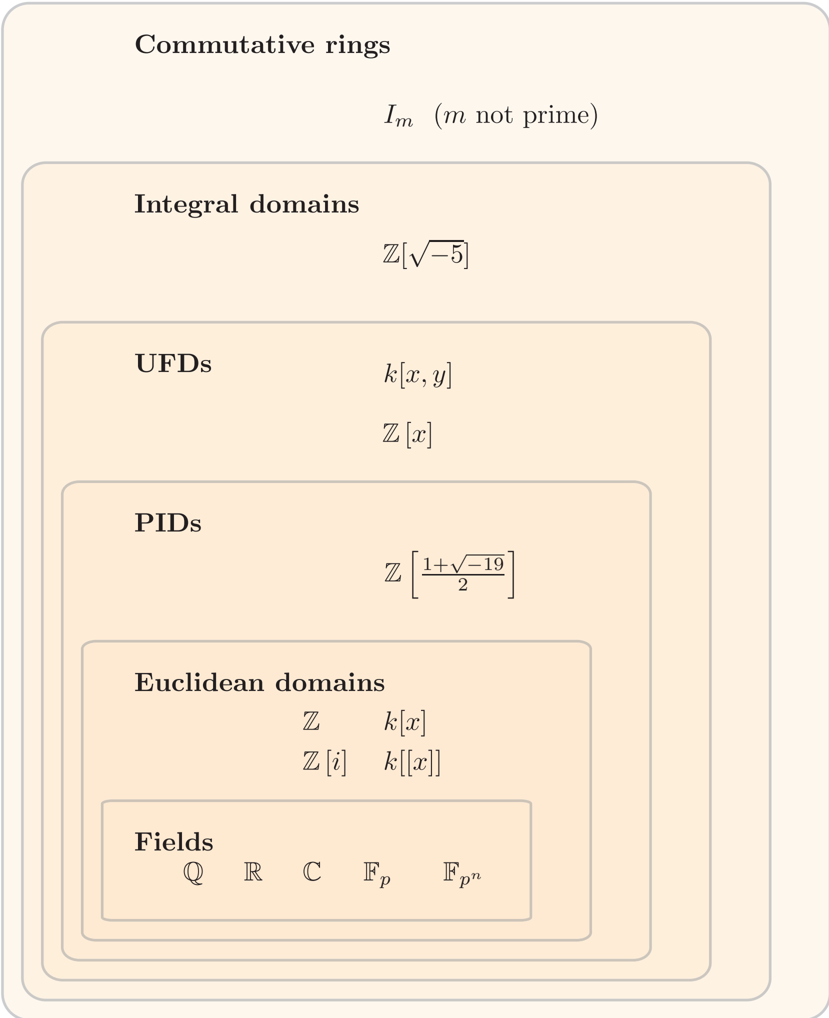

k denotes an arbitrary field in the above diagram.

See Homework 12 for why $\mathbb{Z}[\sqrt{-5}]$ is an integral domain but not a UFD.

The reasons why $\mathbb{Z}[x]$ is located as shown are that it inherits the property of being a UFD from $\mathbb{Q}[x]$ (that's the idea; the details take a little work), and is not a PID because the polynomials with even constant term form an ideal that is not principal.

It takes some work to explain why $\mathbb{Z}\left[\frac{1+\sqrt{-19}}{2}\right]$ is a PID but not a Euclidean domain. See this series of guided exercises.

Here are the reasons why $k[x,y]$ is where it is. It is a UFD because the following general result (which we don't prove in this course) applies to $k[x,y] = (k[x])[y]$: if R is a UFD, then so is $R[x]$. It is not a PID because (x,y) is an ideal which is not principal.

$k[[x]]$ is a Euclidean domain where the degree function gives the lowest power of x with non–zero coefficient.

Fields are Euclidean domains where the degree function is identically zero.

TRR