MATH 6310, Homework 11
Due in class 11/13

Please continue to look over §10 and do these (many of which should be fairly straight-forward checks that you have the definitions straight)

- §10.1, questions 10, 11
- §10.2, question 6
- §10.3, question 11 (see question 9 for the definition of *irreducible*), 27
- §10.4, questions 2, 5, 11, 12, 16

and the following question (for which thanks to Ken Brown). For the definition of *adjoint functors*, see for example Section 8 of the notes on category theory by Ken Brown posted on the course homework page.

1. Let M be a right R-module, N a left R-module, and L a $\mathbb{Z}$-module (i.e., an abelian group).

   (a) Explain how the left action of R on N induces induces a right action of R on the abelian group $\mathrm{Hom}_{\mathbb{Z}}(N, L)$, making the latter a right R-module. (Just give the main points; you don't have to write down every detail.)

   (b) Show that $\mathbb{Z}$-bilinear R-balanced maps $M \times N \to L$ are in 1-1 correspondence with right R-module maps $M \to \mathrm{Hom}_{\mathbb{Z}}(N, L)$. Deduce that there is an abelian group isomorphism
   $$\mathrm{Hom}_{\mathbb{Z}}(M \otimes_R N, L) \;\cong\; \mathrm{Hom}_R(M, \mathrm{Hom}_{\mathbb{Z}}(N, L)).$$

   (c) The result of (b) says (except for one missing detail) that for fixed N, the functor $- \otimes_R N$ from right R-modules to abelian groups is left adjoint to the functor $\mathrm{Hom}_{\mathbb{Z}}(N, -)$ from abelian groups to right R-modules. What is the missing detail?

   (d) If R is commutative, state a result similar to (b) with all Homs being over R.

Read on in §10.5, §11, §12.