## MATH 6310, Homework 12 <br> Due in class 11/27

Please continue to look over $\S 10.5, \S 11, \S 12, \S 15.1$ and do these:

- $\S 10.5$, questions $2(\mathrm{~b}), 6,20,27$
- $\S 11.2$, question 38
- $\S 11.5$, questions 1,9
- §12.2, question 9
- $\S 12.3$, question 15
- §13.6, question 11.

We are unlikely to have gone over Rational Canonical Form and Jordan Canonical Form in class by the time you come to do these exercises. Please fill in the gaps by reading §12. We won't have much time for that chapter in class.

Also please do -

1. Suppose $R$ is a principal ideal domain.
(a) Show that any submodule of a free $R$-module is free.
(b) Show that an $R$-module is projective if and only if it is free.
2. Suppose $V$ is a vector space over a field $F$ and has basis $v_{1}, \ldots, v_{n}$. Suppose $\phi: V \rightarrow V$ is a linear transformation.
(a) Explain why $\phi$ induces a linear transformation $\bigwedge^{n}(V) \rightarrow \bigwedge^{n}(V)$ with

$$
\phi\left(v_{1} \wedge \cdots \wedge v_{n}\right)=\phi\left(v_{1}\right) \wedge \cdots \wedge \phi\left(v_{n}\right)
$$

(b) Sketch a proof that the linear transformation of part (a) is multiplication by $\operatorname{det}(\phi)$.

You can find this result in the textbook: it is Proposition 38 on page 450. I think it is a worthwhile exercise to extract or devise a self-contained proof.

