# MATH 6310, Homework 8
## Due in class 10/23

Please look over §8.3 and on into §9 and do

- §8.3, questions 3 (see Corollary 19 on page 291), 6, 8
- §9.1, question 8
- §9.3, question 4

and —

1. Let $R$ be a UFD with field of fractions $F$. Let $f$ be a monic polynomial in $R[x]$. Show that every root of $f$ in $F$ lies in $R$. (*This problem generalizes Euclid's theorem that* $\sqrt{2}$ *is irrational. It has a straightforward direct proof, but you might find it instructive to deduce it from Gausss lemma.*)

2. Let $R$ be a UFD with field of fractions $F$. Prove that the quotient group $F^\times/R^\times$ is a free abelian group; in other words, it is isomorphic to a direct sum of copies of $\mathbb{Z}$.

3. (a) An integral domain is a UFD if and only if it satisfies:
   (1) Every irreducible element is prime.
   (2) The principal ideals satisfy the ACC.
   
   Much of a proof can be found in the arguments I gave in class. You do not have to repeat those arguments, but please determine and explain the missing steps required to prove the result.
   
   (b) Question 4 in §9.3 gives an example of an integral domain that satisfies (1) but is not a UFD. So (2) must fail. Give an explicit example of an infinite ascending chain of principal ideals in this ring.

(*Thanks to Ken Brown for these questions.*)

Read on in §9 and §13.1, §13.2 and §13.4.