MATH 6310, Homework 9
Due in class 10/30

Please continue to look over §9, §13.1, §13.2, §13.4 and §13.5 and do

- §9.4, questions 2, 16
- §9.5, questions 3, 7
- §13.1, question 1
- §13.2, questions 1, 7

and these two questions (thanks to Ken Brown for the first) —

1. By the universal $n \times n$ matrix I mean the matrix $X = (x_{ij})$ in the polynomial ring $R := \mathbb{Z}[x_{11}, \ldots, x_{nn}]$ in n^2 variables. Show that the determinant of X is irreducible in R. (*This is intuitively clear, since were there a universal factorization of the determinant, you would have learned about it in your first linear algebra course. For a rigorous proof, view* $\det X$ *as a polynomial in* x_{11} *with coefficients in the polynomial ring in the other variables, and apply Gausss lemma. If this seems too trivial, you're probably forgetting to check something.*)
2. Look ahead to §15 to see what is meant by *an algebraic set over a field*. Explain why it follows from Hilbert's Basis Theorem that every algebraic set over a field is the set of common roots of finitely many polynomial equations. (*Thus an infinite collection of equations can always be "replaced" by a finite collection.*)

Read on in §10.