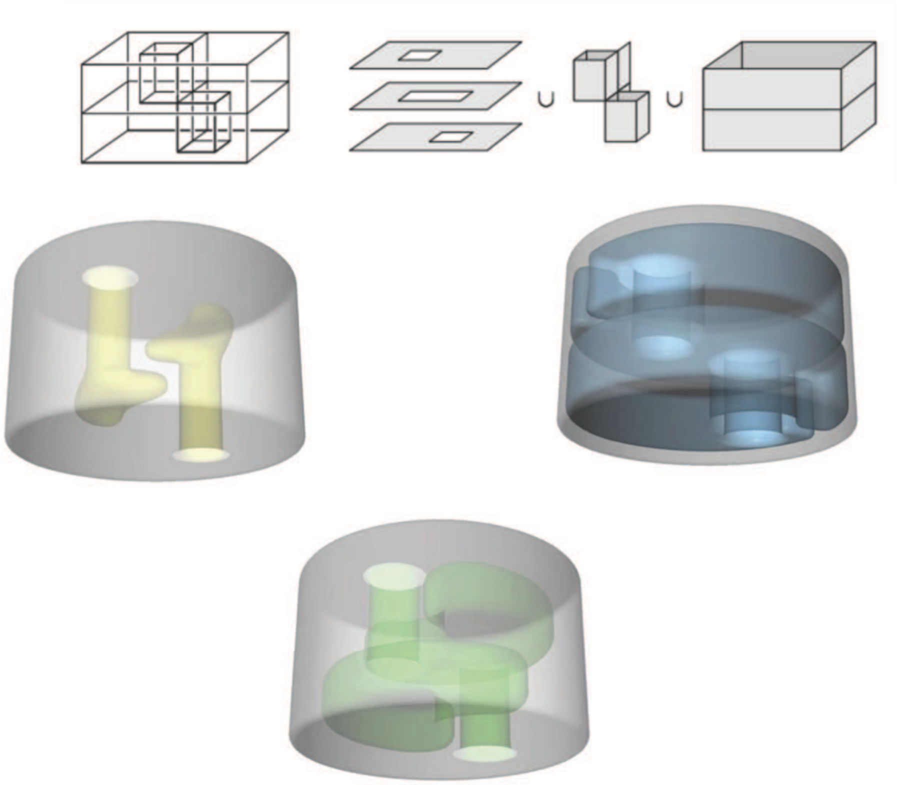
MATH 6510, Algebraic Topology, Spring 2017
Homework 1, Due in class 30 January

Reading. *Bing's House with Two Rooms* is a space which is contractible but not obviously so. Read about it in Chapter 0 of Hatcher, where it is defined as a 2-dimensional subspace of $\mathbb{R}^3$ which is assembled from three parts as shown top right in the figure below; a transparent version is shown top left, and below are some pictures (from sketches of topology) that should help you see that a small neighbourhood of Bing's House is homeomorphic to a ball. You might enjoy this Minecraft construction of (a small neighbourhood of) Bing's House by 'Kid' Hitchman. Perhaps the reason it is hard to see Bing's House is not contractible is that it is not *collapsable*, which is an 'elementary' combinatorial analogue of contractible defined by Whitehead. A few more links: Scientific American (Evelyn Lamb), Biography of R H Bing (wherein we learn that Bing's first names were just 'R' and 'H').



Review the details in Hatcher, Section 1.1 of why the fundamental group is genuinely a group (page 27) and is a homotopy invariant (page 37).

Exercises.

1. Review the definitions of *cone*, *suspension* and *join* in Chapter 0 of Hatcher.
   (a) Prove that the cone CS^{n-1} is homeomorphic to D^n.
   (b) Prove that the suspension ΣS^{n-1} is homeomorphic to S^n.
   (c) Prove that the join $S^m * S^n$ is homeomorphic to S^{m+n+1}. (*Hint: you might like to do the case $m = n = 1$ first as a warm up.*)

2. (Weintraub Ex. 1.3.2.) Let H be the "southern hemisphere" in S^n, $H = \{(x_1, \ldots, x_{n+1}) \in S^n \mid x_{n+1} \leq 0\}$. Let p be the "south pole" $p = (0, 0, \ldots, 0, -1) \in S^n$. Show that the inclusion $(S^n, p) \to (S^n, H)$ is a homotopy equivalence of pairs.

3. Hatcher, page 18, Question 6.

4. Hatcher, page 38, Question 1.

5. Hatcher, page 38, Question 5.

6. Hatcher, page 39, Question 10.