

Of course we expect that, for the most part, your homework solutions will be correct. But simple correctness is not enough. Mathematics must be communicated, and the way you present a solution deeply affects how correct it appears to be. More importantly, the effort to communicate mathematical ideas effectively can be a very useful part of arriving at a correct solution. One of your goals this semester should be to sharpen your presentation.

1. SUBMISSION

Each student is expected:

- to submit her/his own individual write-up of the solutions on time each week;
- to provide a neat copy, stapled in the correct order if there are multiple pages;
- to indicate, on the top of each page, her/his name, the page number, and the number and due date of the assignment;
- to indicate on the top of the cover page fellow students with whom she/he collaborated. Both typed and handwritten assignments are acceptable.

I encourage you to work with your classmates on this homework. Please list your collaborators on this cover sheet. (Your grade will not be affected.) Even if you work in a group, you should attempt the problems yourself before meeting with your group. You must write up your solutions yourself! You should include all computational details, and proofs should be carefully written with full details. These problems will be assessed for completeness. As always, please write neatly and legibly.

2. STYLE

The solution to each exercise should be clearly labeled and contiguous within the write-up, and written in complete sentences with proper spelling, grammar, and punctuation. If an exercise requires a single computation, then, beyond showing the computation, no further text is necessary. There are, roughly speaking two kinds of processes you will need to convey in your write-ups: **computation** and **logical reasoning**. You should also be mindful to portray the **context** in which both of these occur. If an exercise requires multiple computations, for example, then you should indicate the relationships between them (e.g., which is done first, or which, if any, depends on the other, etc.). If it asks why a certain statement is true, then you should write a complete sentence, possibly restating the assertion, and indicating the connection between the statement and the reason you give.

3. TWO EXAMPLES

Suppose an exercise in the text says, "Compute the following products of complex numbers, and simplify the expression," and it has three parts. Your homework solution should look something like this:

p. 12, #4: Complex multiplication

(a) $(1 + 2i)(-2 + i) = -4 - 3i.$

(b) $(1 - 3i)^3 = 1^3 - 3(1)^2(3i) + 3(1)(3i)^2 - (3i)^3 = 26 + 8i.$

(c) $4(a + bi) = 4a + 4bi.$

When you are asked to prove a statement or to show that it is true, this means give a convincing and logical argument that the statement is true. In your write-up, you should either write the full statement of the exercise, or simply write the statement to be proved. Then, give your argument. For example, suppose you are told, "Show that the derivative of a polynomial is a polynomial." Then, you might write the following:

p. 35 #17: The derivative of a polynomial is a polynomial.

Proof. Let $f(x) = a_0 + a_1x + \cdots + a_nx^n$ be a polynomial. By the sum rule for derivatives, we can differentiate each term separately. The derivative of a_0 is zero. By the power rule, $(x^k)' = kx^{k-1}$ for each $k \geq 1$. Also, since each a_k is a constant coefficient, $(a_kx^k)' = ka_kx^{k-1}$. Therefore, the derivative of f is given by

$$f'(x) = a_1 + 2a_2x + \cdots + na_nx^{n-1},$$

which is a polynomial. □

Some things to keep in mind when writing mathematical arguments:

- you should deal with the most general case that applies;
- there will often be special cases as well (e.g., in the above example, the case of the constant polynomial $f(x) = a_0$ is handled separately).
- you should define and use clear and concise notation;
- you should state and use known results to reach the desired conclusion.

Writing proofs takes practice, and this course is intended in part to give you an opportunity to practice and thereby develop your communication skills in mathematics.

4. THE WRITING PROCESS

Communicating mathematics is a skill that will transfer to other quantitative disciplines. Composing a logical argument is valuable in every career. I encourage you to take advantage of the feedback you get this semester, from peers, the TAs and me, to hone your writing skills. Writing any piece is a process. Your first scratchings at solving a problem will barely resemble your final written solution. What you hand in should represent your best effort to communicate how you solved a problem, distilled to clear well-reasoned prose.

An anonymous student recently reflected on her own writing process when attempting a problem set [1]. Her description of her problem-solving and writing process is a model to emulate. Note that she typesets her solutions in \LaTeX . I do not expect you to do so, but will provide resources on the course web page should you be interested in learning about this typesetting software. (It beats the pants off of typing math in Microsoft Word.)

A chronological checklist

A) I read the problems the day I get them or soon thereafter. I usually have to read them a lot of times before I even understand what is going on in them. But half the battle is knowing what's going on, so I try to get that done as soon as I can. I look for words I've heard before and think about whether I've seen anything similar. I usually write a "translation" to the side of the problem, something that helps me remember what the problem is asking when I go back to it later.

B) I scrawl. I write ideas down, even if they're stupid (and they probably are at first) or messy. They are usually very unorganized. I scrawl all over the homework

paper and when I run out of room there, I scrawl all over notebook paper. This is just to get my ideas out, and if I don't have any ideas, I write out definitions that might be relevant, or really anything at all that might be relevant. For example, if the problem is to show two sets are equal, I write down what I know has to be shown for that to be true: containment in both directions. Just by writing that down and seeing it, I might think of the next step. For that matter, sometimes I just write down the entire problem in my own words.

C) I organize my scrawlings. Still on notebook paper, I write the problems out in order and compile all the ideas I had for each problem into one place. The answers might not be complete, but the ideas are at least organized.

D) I get frustrated and leave it alone for a while. Depending on how much time is left, I stop thinking about it for a few hours or a day so I don't burn out. These homeworks are really hard and if I think about it too much all at once I start getting mad and thinking "When am I ever gonna use this stuff???" Not productive. I take a break and do something completely different.

E) I come back fresh. I take my organized scrawlings to the math lab and crank it out. If no ideas ever came, I ask whoever is around if any ideas ever came to them, including Patrick, who lives right across the hall from the math lab, conveniently! I write out a dress rehearsal of my homework (the whole thing the way I want it to look Latexed, just on notebook paper).

F) I \LaTeX it. I won't lie, this takes me forever. But I'm getting better at it and it comes much much easier than it did at first. I usually copy and paste an old homework into a new document and fill things in. That way, I don't have to start from scratch. \LaTeX ing it makes it SO much clearer and I can find mistakes more easily.

G) I revel in the beauty that \LaTeX spits out. So lovely!

That's it mostly. It does take a lot of work but it feels so good to turn in a complete, correct, \LaTeX ed answer. That is, it's worth it.

Good luck! I have confidence that you'll succeed at making as much out of this opportunity as possible.

REFERENCES

[1] *Helpful hints, volume 1*. Web

<http://changeofbasis.blogspot.com/2009/03/helpful-hints-volume-1.html>.

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