4130 HOMEWORK 3

Due Thursday February 18

(1) Let $\{x_n\}$ and $\{y_n\}$ be Cauchy sequences of rational numbers. Prove that $\{x_n\} \sim \{y_n\}$ if and only if for all $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that for all $m, n > N$, $|x_m - y_n| < \varepsilon$.

(2) (a) Using the formula for the partial sums of a geometric series, or otherwise, check that the sequence of rational numbers whose n^{th} term is

$$x_n = \sum_{i=1}^{n} \frac{e_i}{10^i}$$

is a Cauchy sequence, for any integers e_i with $0 \leq e_i \leq 9$. (Remark: the real number defined by this Cauchy sequence is denoted $0.e_1e_2e_3\ldots$)

(b) Show that $0.999\ldots = 1$.

(3) Section 2.2.4 #5.

(4) Using the triangle inequality, show that for any $a, b \in \mathbb{R}$, we have

$$||a| - |b|| \leq |a - b|.$$

Using this, show that if $\{x_n\}$ is a sequence of real numbers which converges to L, then $\{|x_n|\}$ converges to $|L|$.