4130 HOMEWORK 4

Due Tuesday March 2

(1) Let $\mathbb{N}^{\mathbb{N}}$ denote the set of all sequences of natural numbers. That is,

$$\mathbb{N}^{\mathbb{N}} = \{(a_1, a_2, a_3, \ldots) : a_i \in \mathbb{N}\}.$$

Show that $|\mathbb{N}^{\mathbb{N}}| = |\mathcal{P}(\mathbb{N})|$.

(2) Let $\{x_n\}$ be a Cauchy sequence of rational numbers. Regarding $\{x_n\}$ as a sequence of real numbers, show that $\{x_n\}$ converges to the real number x defined as the equivalence class of the sequence $\{x_n\}$.

(3) Section 2.2.4 # 4.

(4) Show that every subset S of $\mathbb{R}$ which is bounded below has a greatest lower bound. (Hint: see p. 75 of the textbook.)

(5) Find, if they exist, the supremum (least upper bound) and infimum (greatest lower bound) of the following subsets of $\mathbb{R}$.
   (a) $\{1, 2, 3\}$.
   (b) $(0, 1) \cup \{2\} \cup [3, 4) = \{x \in \mathbb{R} : 0 < x < 1 \text{ or } x = 2 \text{ or } 3 \leq x < 4\}$.
   (c) $\{1 - \frac{1}{n} : n \in \mathbb{N}\}$.
   (d) $\mathbb{Q}$.

(6) Prove Theorem 2.3.2 in the textbook.