# 4130 HOMEWORK 7

**Due Tuesday April 13**

(1) Let $D \subset \mathbb{R}$. Let $f, g : D \to \mathbb{R}$ and let $a$ be a cluster point of $D$. Suppose $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$. Show that $\lim_{x\to a} f(x)g(x) = LM$.

(2) Section 5.2.4 Exercise 3.

(3) Section 5.2.4 Exercise 4.

(4) Let $U$ be an open subset of $\mathbb{R}$ and suppose $f : U \to U$ is a $C^1$ bijection and that $f'(x) \neq 0$ for all $x \in U$.

   (a) Use the Inverse Function Theorem to show that the inverse function $f^{-1} : U \to U$ is also $C^1$.

   (b) Give an example to show that $f^{-1}$ may not be $C^1$ if $f'(x) = 0$ for some $x \in U$.

(5) Section 5.4.6 Exercise 22(b). (Hint: let $y = p_1x_1 + \cdots + p_nx_n$. For each $1 \leq i \leq n$, estimate $f(x_i)$ using the Taylor expansion of $f$ about the point $y$, applying the Lagrange Remainder Theorem from page 188.)