4130 HOMEWORK 8

Due Tuesday May 3

(1) Let $f_n : A \to \mathbb{R}$ be functions which converge uniformly on A to a function f. Let x_0 be a cluster point of A. Suppose $\lim_{x\to x_0} f_n(x)$ exists for all n. Let $L_n = \lim_{x\to x_0} f_n(x)$.
   (a) Show that the sequence $\{L_n\}$ converges.
   (b) Show that $\lim_{x\to x_0} f(x)$ exists and equals $\lim_{n\to\infty} L_n$.

(2) Section 7.3.4 Exercise 11.

(3) Find the radius of convergence of the power series

$$f(x) = \sum_{n=0}^{\infty} (n^2 + n + 1)x^n.$$

Find a pair of polynomials $p(x)$ and $q(x)$ such that $f(x) = \frac{p(x)}{q(x)}$ within its radius of convergence.