

Quadratic Surfaces

Math 1920

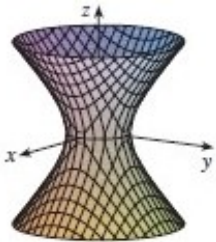
1. Ellipsoid



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- Horizontal traces: Ellipses.
- Vertical traces: Ellipses.
- Put $a = b = c$ and you get a sphere.

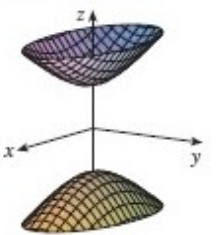
2. One-sheeted hyperboloid



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

- Horizontal traces: Ellipses.
- Vertical traces: Hyperbolas.

3. Two-sheeted hyperboloid



$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- Horizontal trace in $z = k$ is an ellipse, but only if $k^2 \geq c^2$.
- Vertical traces: Hyperbolas.

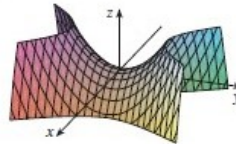
4. Elliptic paraboloid



$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

- Horizontal traces: Ellipses.
- Vertical traces: Parabolas.

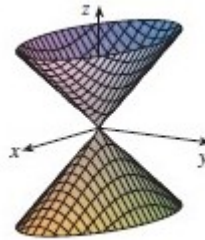
5. Hyperbolic paraboloid



$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

- Horizontal traces: Hyperbolas.
- Vertical traces: Parabolas.

6. Elliptic cone



$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

- Horizontal traces: Ellipses.
- Vertical traces: Hyperbolas, except the trace in $x = 0$ or $y = 0$, where the trace is a pair of lines.