Homework set IV

Question 1. Find Iwasawa decomposition (KAN–decomposition) and the KAK -decomposition of Lie groups $SL_n(\mathbb{R})$ and $SO(p,q)$.

Question 2. Let Γ be a lattice of a closed linear group G, and let X be the quotient $\Gamma\backslash G$. Show that for any $x \in X$, there is a positive number r (so called the injectivity radius at x) such that $B_r^G \to B_r^X(x)$, $g \mapsto xg$ is an isometry.

Question 3.

1. Show that the group $SL_2(\mathbb{R})$ is generated by two elements τ and σ, where τ, σ are 2×2 matrices with entries $1, 1, 0, 1$ and $0, -1, 1, 0$, resp. Find a presentation of $PSL_2(\mathbb{R})$ with these generators.
2. Show that $SL_2(\mathbb{Z})$ is a lattice of $SL_2(\mathbb{R})$ by computing the area of the fundamental domain $\{z \in \mathbb{C} : |z| \geq 1, Re(z) \leq 1/2\}$.

(Reference : Knapp, Lie groups beyond introduction, Eisiedler-Ward, Chapter 15)