Homework set V

Question 1. Let f be a continuous function $f : X \to X$ of a compact topological space X. For any integer k, show that $h_{top}(f^k) = k h_{top}(f)$.

Question 2. A function $f : X \to X$ is called expansive if there exists an $\epsilon_0 > 0$, called the expansiveness constant, such that for all $x \neq y$, $\sup d(f^n(x), f^n(y)) \geq \epsilon_0$. Show that $h_{top}(f) < \infty$ if f is expansive. Show also that $h_{top}(f) = \lim \frac{1}{n} \log N_{d_n^f}(\epsilon)$, for all $\epsilon < \epsilon_0$.

Question 3. Let $v = (a, b)$, $a/b \notin \mathbb{Q}$. For a continuous function $f : \mathbb{T}^2 \to \mathbb{R}$, and $x \in \mathbb{T}^2$, show that

$$\lim_{T \to \infty} \frac{\int_0^T f(\varphi_t(x))dt}{T} = \int_{\mathbb{T}^2} f d\mu.$$