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Some of you were asking after class if there is a more general Mean Value theorem.

In one way, this would involve finding a $\vec{c}$ satisfying the mean value theorem for pairs of vectors $(\vec{a}_i, \vec{b}_i)$ simultaneously, and while there is a solution $\vec{c}_i$ for any interval $[\vec{a}_i, \vec{b}_i] \subseteq U$, there is no reason to believe that we can use the same $\vec{c}$ for all the pairs simultaneously. That is the sense in which the Mean Value Theorem is as general as possible.

It is possible to give a generalization of Rolle's Theorem in the multi-dimensional setting. One thing that is true is the following.

> PROP: Suppose $f: \mathbb{R}^n \to \mathbb{R}$ is continuous. Then for any real number $r > 0$ and $x_0 \in \mathbb{R}^n$, f attains a maximum (or minimum) value on the domain $D_r(\vec{x}_0) = \{ \vec{x} \in \mathbb{R}^n \mid \|\vec{x} - \vec{x}_0\| \leq r \}$.
> Moreover, if $\vec{c} \in D_r(\vec{x}_0)$ is a point where f attains an extreme value, and if f is differentiable at $\vec{c}$, then $[Df(\vec{c})]$ is the $1 \times n$ 0-matrix.

One can use this, and other results, to prove a multidimensional version of Rolle's theorem. See the paper about this, linked from the course website.