NOTES AFTER LECTURE ON 30 AUGUST 2007
~~MATH 223.~~

The cross-product.

We can think of the cross-product as a function

$$\times : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$
$$(\vec{v}, \vec{w}) \longmapsto \vec{v} \times \vec{w}.$$

It is possible to prove (can you?!) that the cross-product satisfies

1. $\vec{v} \times \vec{w} = -\vec{w} \times \vec{v} \quad \forall \vec{v}, \vec{w} \in \mathbb{R}^3$;
2. $(a\vec{u} + b\vec{v}) \times \vec{w} = a(\vec{u} \times \vec{w}) + b(\vec{v} \times \vec{w}) \quad \forall \vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^3$ & $\forall a, b \in \mathbb{R}$;

and 3. $\vec{u} \times (\vec{v} \times \vec{w}) + \vec{v} \times (\vec{w} \times \vec{u}) + \vec{w} \times (\vec{u} \times \vec{v}) = \vec{0} \quad \forall \vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^3$.

These three properties tell us that $\mathbb{R}^3$, together with this cross-product, is a <u>Lie algebra</u>.

Lie, which rhymes with see, is the name of a Norwegian mathematician who first realized that it is useful to study the now-called Lie algebras when trying to study related gadgets now called Lie groups. Lie theory is currently an active and important area of mathematical research.