Instructions for extra credit: You can earn extra credit this week by handing in solutions to all of the starred exercises, the additional exercises, and five other problems of your choice from the problems listed below. Your problem set should be turned in in class on 29 November 2007.

In any case, these problems should be useful as you start to review for the final exam. Good luck studying, safe travels, and have a happy Thanksgiving!

Reading. §3.8.

Problems from the book.

- §1.10: 2, 7, 22, 26⋆, 28, 31.
- §2.11: 3, 5, 8, 11, 30⋆.
- §3.8: 1, 2⋆, 6.
- §3.9: 4⋆, 6.

Additional problems.

1. Recall from lecture that a **variety** is the zero set of a (polynomial) function f, and the tangent space to a point x in the variety is defined to be $\ker[Df(x)]$. The point is non-singular if $[Df(x)]$ is onto, and singular otherwise. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by
$$f\begin{pmatrix} x \\ y \end{pmatrix} = y^2 - x^3.$$
Consider the variety X that is the zero set of this curve. Compute the tangent space to the point $\begin{pmatrix} a^2 \\ a^3 \end{pmatrix}$ for all real numbers a. Is the variety singular at any of these points?
2. Given an example of a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that the surface defined by $f(x) = 0$ has tangent space that is two-dimensional at all points, except for a curve on the surface where the tangent space is three-dimensional.