DUE: 30 October 2007

Reading. §2.10, §§3.1–3.3.

Problems from the book: the starred problems may be carefully graded. You will be given a "completeness" score, depending on how carefully you complete these problems.

- §2.10: 1abc, 2, 3, 8, 9, 13, 16*
- §3.1: 1, 2, 8*, 9, 19
- $\S 3.2: 1, 4^*, 5, 6^*$

Additional problems: These will be carefully graded!

1. Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$f(x,y) = (x^2 - y^2, 2xy).$$

- a. Show that f is one-to-one on the set A consisting of all (x,y) with x>0. [HINT: If $f(x,y)=f(\alpha,b)$, then $|f(x,y)|=|f(\alpha,b)|$.]
- b. What is the image B = f(A)?
- c. If g is the inverse function, find $\left[Dg\begin{pmatrix}0\\1\end{pmatrix}\right]$.
- 2. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be given by the equation $f(x) = |x|^2 \cdot x$. Show that f is differentiable and that f carries the unit ball centered at $\overrightarrow{0}$ to itself in a one-to-one fashion. Show, however, that the inverse function is not differentiable at $\overrightarrow{0}$.