

## MATH 224 – SPRING 2008 – PROJECTS

Every student must hand in a written project. Your grade on the project will comprise 25% of your final grade. Projects should involve learning the material for a subject **not covered** in class, roughly a week's worth of material. You will then write up an exposition of the mathematics you have learned, including motivation, examples, at least one theorem, and at least one (detailed sketch of a) proof. You must meet with me to get your project approved. The time table for these projects is:

- February 29, 2008: Your topic must be approved.
- March 27, 2008: Outlines due (Thursday after spring break).
- April 17, 2008: If you hand in a rough draft by this date, I will return it within a week with comments and suggestions. There is no grade penalty for this. You are not required, but strongly recommended, to take advantage of this option!
- May 2, 2008: Projects due by **noon**!

### Possible topics.

A non-exhaustive list is given below. You are free to choose a topic not on the list, but you should check with me for approval. (I am aiming for you not to bite off more than you can chew!)

- **The Central Limit Theorem.** Probability distributions tend to the bell curve. We only touched on this in class.
- **The Banach-Tarski Paradox and/or the Hausdorff paradox.** Using very special scissors, you can cut a sphere up into weird pieces with non-intuitive properties.
- **Random walks.** If you take a random walk on the real line, then you always come back. (Random here means that you take steps randomly chosen, with equal probability, to the left or to the right.) Why is this true? What about higher dimensions?
- **Transcendental numbers.** We won't discuss this at all in class, but it could be an interesting topic.
- **Numerical integration.** More on numerical techniques described in detail. You could do some programming here to compare Monte Carlo and other method(s) to compute various integrals.
- **Fractals and fractal dimension.** We will only discuss this briefly in class. One possibility is to do an in depth survey of the basic results in this active field.
- **Fourier series.** Functions that are periodic with period  $2\pi$  can be written as infinite "sums"

$$\sum_{n \geq 0} a_n \cos(nx) + b_n \sin(nx).$$

Understanding how to do this, what the convergence issues are, and other aspects of the decomposition, would make a good project.

- **Gödel's incompleteness theorem.** Studying subtle issues in the foundations of mathematics would be a suitable topic. This might start with the Axiom of Choice, and continue to one or both of the Incompleteness Theorems.

- **Calculus of variations.** This can be used to prove things like the isoperimetric inequality, which relates the length of a simple closed curve to the area that it bounds.
- **Curvature and its other guises.** You could study some basic differential geometry: different notions of curvature, minimal surfaces, and so forth. This would build on material we developed in Math 223.
- **Anything else that interests you.** Come talk to me and propose something!

### Reference information.

You might start by looking on Wikipedia. It contains a wealth of information on many topics, and is a good starting point. It is not always 100% accurate, so you should eventually read some articles and/or books. You can find mathematics articles by using MathSciNet.

From a University computer (say, at the library), go to

<http://www.ams.org/mathscinet/search>

Click on "Author" and select "Journal." Enter "*American Mathematical Monthly*." Then under title, enter a word or two relating to your subject, and click on "Search." This will return articles in the *American Mathematical Monthly*, a mathematics journal aimed at undergrads, with your keyword(s) in the title.