When you hand in this problem set, please indicate on the top of the front page how much time it took you to complete.

Reading. §4.1–4.3.

Problems from the book:

- 4.1.2, 4.1.6, 4.1.10, 4.1.12, 4.1.14, 4.1.18.
- 4.2.1, 4.2.2, 4.2.5.

Additional problems:

1. Fill out the background survey and come by my office to return it to me.
2. Suppose that a meter stick is broken at two randomly chosen points. What is the probability that the three segments form the sides of a triangle?
3. Let $f(x) : [0, 1] \to \mathbb{R}$ be an increasing function. Can you determine whether or not f is integrable on $[0, 1]$? Why or why not?
4. Reread the definitions and properties of abstract vector spaces. Let V be a vector space, and let $W \subseteq V$ be a subspace. For $u, v \in V$, we write $u \sim_W v$ if $u - v \in W$. Look up the definition of the term **equivalence relation** (in another math book or on the internet).
   a. Prove that $\sim_W$ is an equivalence relation. We denote the set of equivalence classes by the symbol V/W.
   b. For $v \in V$, let $[v]$ denote its equivalence class in V/W. Define an addition rule $\tilde{+}$ on V/W by $[u]\tilde{+}[v] = [x + y]$, where $x \in [u]$, $y \in [v]$, and $+$ is the usual addition in V. Prove that this addition on equivalence classes is well-defined, and that it does not depend on the choices of x and y.
   c. Come up with a definition of scalar multiplication on V/W, and show that it is well-defined.
   d. Prove that V/W is a vector space over $\mathbb{R}$.
   e. Let $V = \mathbb{R}^3$ and W those vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$ satisfying $x + y + z = 0$. Prove that $\mathbb{R}^3/W$ is one-dimensional.