Reading. §4.10–4.11.

Problems from the book:

- 4.9.4, 4.9.6
- 4.10.2, 4.10.4, 4.10.5, 4.10.12, 4.10.13, 4.10.14
- 4.11.1, 4.11.2

Additional problems:

1. Compute $\iint_R (x+y)e^{x^2-y^2}\,dA$, where R is the region bounded by the lines $x-y=0$, $x-y=2$, $x+y=0$ and $x+y=3$.

2. Evaluate $\iiint_R xyz\,|dx\,dy\,dz|$ over the region R between the spheres centered at the origin of radius $r=2$ and $r=4$, and above the cone $\phi=\frac{\pi}{3}$.

3. Recall that a matrix is **orthogonal** if and only if $AA^T = I$. Let $O(n)$ denote the set of $n \times n$ orthogonal matrices.

   a. Show that $O(n)$ is a group.

   b. Let $A \in O(n)$ and let $\chi_A(t)$ denote its characteristic polynomial. Suppose that $\alpha \in \mathbb{R}$ is a root of $\chi_A(t)$. Prove that $\alpha = \pm 1$.

   c. We have seen in class that a permutation matrix M_σ is an orthogonal matrix, for every $\sigma \in S_n$. Can you describe all of the roots of the characteristic polynomial of M_σ? Your answer should depend on the cycle type of σ.

   d. We define $SO(n) = \{ A \in O(n) \mid \det(A) = 1 \}$. This is the **special orthogonal group**. Can you give an example of an element of $SO(2)$ such that none of the roots of the characteristic polynomial are real numbers? As best you can, describe $SO(2)$.

4. Let V be a vector space, and $W \subseteq V$ a subspace. Let $T : V \to V$ be a linear transformation satisfying $T(W) \subseteq W$. Show that $\tilde{T} : V/W \to V/W$ defined by $\tilde{T}([v]) = [T(v)]$ is a well-defined linear transformation from V/W to V/W.

5. Let $T : \mathbb{R}^5 \to \mathbb{R}^5$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 - 3x_3 \\ -2x_1 + 3x_2 \\ x_1 - 4x_3 \\ x_4 - 3x_5 \\ 4x_4 + x_5 \end{bmatrix}.$$

   a. Find a three dimensional subspace W of $\mathbb{R}^5$ such that $T(W) \subseteq W$; that is, for each $w \in W$, $T(w) \in W$.

   b. Let $T|_W$ be the restriction of T to W; that is, $T|_W : W \to W$ is a linear transformation from W to itself. Compute $\det(T)$, $\det(T|_W)$, and $\det(\tilde{T})$, where $\tilde{T}$ is defined as in Problem 4. Can you explain the relationship between these determinants (and give a reason why)?