Reading. §5.5, 6.1–6.3.

Problems from the book:

- 5.5.1, 5.5.2
- 6.1.2, 6.1.6, 6.1.8, 6.1.11, 6.1.12, 6.1.16
- 6.2.2, 6.2.6
- 6.3.3, 6.3.6

Additional problems:

1. Cornell University has about 13,500 undergraduate students and 6,300 graduate students. Assuming that each student has three initials, is it true that there must be at least two undergraduate students with the same initials? At least two students (undergrad or grad) with the same initials?
2. Cornell University needs to set up an inter-college committee consisting of two faculty from the Engineering School. three from Arts & Sciences, one from ILR, two from Agriculture, and two from the Vet School. If there are 15 eligible faculty members in each school, how many different possible committees are there?
3. If a rental car company has 95 cars with a total of 465 seats, can we be sure that there is a car with at least 5 seats?
4. If a telephone switching network of 20 switching stations averages 65,000 connections for each station, what can you say about the number of connections in the smallest station? What can you say if you know that the largest switching station has no more than 68,000 connections?
5. Let U, V, and W be vector spaces, and $T : U \to V$ and $S : V \to W$ linear transformations. We call this set of transformations and vector spaces **exact (at V)** if
$$\ker(S) = \operatorname{im}(T).$$
This is usually called an **exact sequence**.
   a. Give, explicitly, an exact sequence where U, V, and W have dimensions 3, 4, and 5 respectively. (Give explicit matrices for S and T.)
   b. Prove that if T is injective and S is surjective, and the sequence is exact, then
$$\dim(U) + \dim(W) = \dim(V).$$