Math 4310

Homework 1

Due 8/29/12

Name: ______________________________

Collaborators: ______________________________

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Please print out these pages. I encourage you to work with your classmates on this homework. Please list your collaborators on this cover sheet. (Your grade will not be affected.) Even if you work in a group, you should write up your solutions yourself! You should include all computational details, and proofs should be carefully written with full details. As always, please write neatly and legibly.

Please follow the instructions for the "extended glossary" on separate paper (LaTeX it if you can!) Hand in your final draft, including full explanations and write your glossary in complete, mathematically and grammatically correct sentences. Your answers will be assessed for style and accuracy.

Please **staple** *this cover sheet, your exercise solutions, and your glossary together, in that order, and hand in your homework in class.*

GRADES

Exercises ______________________ / 50

Extended Glossary

Component	Correct?	Well-written?
Definition	/6	/6
Example	/4	/4
Non-example	/4	/4
Theorem	/5	/5
Proof	/6	/6
Total	/25	/25

Exercises.

1. Use induction to verify the following statements.

   (a) For any integer $n \geq 1$, $1^2 + 2^2 + \cdots + n^2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

   (b) Recall that the **Fibonacci numbers** are defined by $f_1 = f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for values of $n \geq 3$. So the Fibonacci sequence is $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots$.

   For any integer $n \geq 1$, we have $\sum_{k=1}^{n} (f_k)^2 = f_n \cdot f_{n+1}$.

   (c) For any integer $n \geq 1$, the Fibonacci numbers satisfy $f_1 + f_3 + \cdots + f_{2n-1} = f_{2n}$.

2. (**Curtis, p. 15 #4**) Let $\mathbb{F}_2$ denote the set consisting of two elements $\{0, 1\}$, with the operations defined by the tables

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

   Show that $\mathbb{F}_2$ is a field, with the additional property that $2\alpha = \alpha + \alpha = 0$ for all $\alpha \in \mathbb{F}_2$.

3. Is the set $\{ a + b\sqrt{7} \mid a, b \in \mathbb{Q} \}$ a subfield of $\mathbb{R}$? If so, please provide a proof. If not, explain why not.

Extended Glossary. Please give a definition of a **perfect square**. Then give an example of a perfect square, an example of a number that is not a perfect square, and state and prove a theorem about perfect squares.

You may work in groups, but please write up your solutions **yourself**. If you do work together, your group should come up with at least two examples, two non-examples, and two theorems. Each one (example/non-example/theorem) should be included in some group member's extended glossary. Your solutions should be written formally, so that we could cut and paste them into a textbook.