The Greek mathematician Pythagoras is often credited with proving the following theorem. We give a proof that is essentially a single figure. We include the full details explaining why this figure establishes the Pythagorean Theorem.

The Pythagorean Theorem. *Consider a right triangle with leg lengths* a *and* b*, and hypotenuse length* c*. These lengths satisfy*

$$a^2 + b^2 = c^2.$$

Proof. We may arrange four copies of the right triangle into a square, as shown in the figure below. We

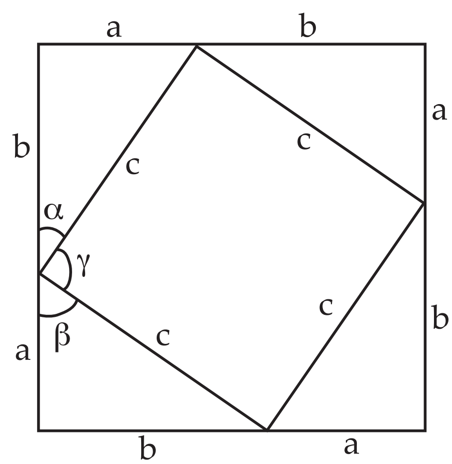

FIGURE 1. Four copies the right triangle, arranged in a square.

notice that the angle marked γ is a right angle, since $\alpha + \beta + \gamma = 180°$, and the angles α and β are the two acute angles in a right triangle, so their sum is $\alpha + \beta = 90°$. Therefore, $\gamma = 90°$, and the quadrilateral in the center of the figure must be a square.

The entire square has side lengths $a + b$, and the central square has side length c. We may compute the area A of the entire square as $A = (a + b)^2$, or by adding up the areas of the four triangles plus the area of the central square,

$$A = 4 \cdot \left(\frac{1}{2}ab\right) + c^2.$$

Thus we have

$$(a + b)^2 = 2ab + c^2.$$

We now multiply out the left-hand side to get

$$a^2 + 2ab + b^2 = 2ab + c^2.$$

Subtracting $2ab$ from each side, we get the desired equality, $a^2 + b^2 = c^2$. □