Math 4310

Homework 1

Due 9/2/15

Name: _____

Collaborators: _____

_____

Please print out these pages. I encourage you to work with your classmates on this homework. Please list your collaborators on this cover sheet. (Your grade will not be affected.) Even if you work in a group, you should write up your solutions yourself! You should include all computational details, and proofs should be carefully written with full details. As always, please write neatly and legibly.

Please follow the instructions for the "extended glossary" on separate paper (LaTeX it if you can!) Hand in your final draft, including full explanations and write your glossary in complete, mathematically and grammatically correct sentences. Your answers will be assessed for style and accuracy.

Please **staple** *this cover sheet, your exercise solutions, and your glossary together, in that order, and hand in your homework in class.*

Exercises.

1. Please prove the following statements.

    (a) An integer n is even if and only if its square n^2 is even.

    (b) Recall that the **Fibonacci numbers** are defined by $f_1 = f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for values of $n \geq 3$. So the Fibonacci sequence is $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots$. For any integer $n \geq 1$, we have

    $$f_1 + f_2 + \cdots + f_n = f_{n+2} - 1.$$

    (c) For any integer $n \geq 2$, the Fibonacci numbers satisfy

    $$f_n^2 - f_{n-1} \cdot f_{n+1} = (-1)^{n-1}.$$

2. Use the Euclidean algorithm to find the following greatest common divisors.

    (a) $\gcd(252, 180)$

    (b) $\gcd(513, 187)$

    (c) $\gcd(7684, 4148)$

3. Prove the following facts about divisors. In the following, $a, b, c \in \mathbb{N}$.

    (a) If $b|a$, then $b|ac$.

    (b) If $b|a$ and $c|b$, then $c|a$.

    (c) If $c|a$ and $c|b$, then $c|(ma + nb)$ for any integers m, n.

Extended Glossary. Please give a definition of a **prime number**. Then give an example of a prime number, an example of a number that is not a prime number (don't forget to explain why!), and state and prove a theorem about prime numbers.

You may work in groups, but please write up your solutions **yourself**. If you do work together, your group should come up with at least two examples, two non-examples, and two theorems. Each one (example/non-example/theorem) should be included in some group member's extended glossary. Your solutions should be written formally, so that we could cut and paste them into a textbook.