Math 4320 Name: ______________________________

Prelim Part 1 4 March 2015 Time: 50 minutes

INSTRUCTIONS — PLEASE READ THIS NOW

• You will be graded on all the True/False questions, and on the **two** problems you choose of the three remaining problems. Your score will be out of a possible total of 40 points. **On this cover sheet, please circle** the problem numbers for which you wish us to grade. If you do not circle any problems on this cover sheet, we will grade the first three problems you have started to answer.

• Please carefully write all your final answers on the page they are posed. You should include a complete logical justification, and you answers should be written in grammatically correct mathematical language. There are 2 extra blank pages at the end of the exam, but we **will not read** your work on those pages.

• Write your name on this sheet **right now**.

• Look over your test packet **as soon as the exam begins**. If you find any missing pages or problems please ask for another test booklet.

• You have 50 minutes to complete this exam. You may leave early, but if you finish within the last 10 minutes, please remain in your seat.

• This is a closed book exam. You are **NOT** allowed to use a calculator, cell phone, or any other electronic device (not even as a time keeping device).

• **Academic integrity** is expected of every Cornell University student at all times, whether in the presence or absence of a member of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination. I will not discuss this exam with other students until both sections have taken the exam.

Please sign below to indicate that you have read and agree to these instructions.

____

Signature of Student

OFFICIAL USE ONLY

T/F	______	/ 10	______
1.	______	/ 15	______
2.	______	/ 15	______
3.	______	/ 15	______
Total:	______	/ 40	______

True/False. (2 points each) Please circle **TRUE** if the statement is always true, or **FALSE** if it fails in at least one example. You do **not** need to justify your answer, and I will not read what you write in the spaces below.

(a) The cycles $(7, 6, 5)$ and $(3, 58, 28)$ are conjugate in S_{100}.

TRUE	FALSE

(b) The subgroup $\langle(1, 2, 3)\rangle$ is a normal subgroup of S_4.

TRUE	FALSE

(c) The set of all positive real numbers with operation multiplication is a group.

TRUE	FALSE

(d) The function $f : S_3 \to \mathbb{I}_3$ sending each element to its order $f(\alpha) = \mathrm{ord}(\alpha)$ is a homomorphism.

TRUE	FALSE

(e) The function $\mathrm{Sq} : (\mathbb{R}, +) \to (\mathbb{R}, +)$ defined by $\mathrm{Sq}(x) = x^2$ is a homomorphism.

TRUE	FALSE

Please answer **two** of the following three questions. The questions are repeated, with space for your answers, on the following pages. **On the cover sheet,** please circle the problems you would like us to grade. If you do not circle any problems on the cover sheet, we will grade the first two problems you have started to answer.

Question 1. Equivalence relations. $(3+6+6$ points per part)

(a) Give the definition of an **equivalence relation** on a set X.

(b) Let $\sigma : X \to X$ be a bijection. Define a relation on X by $x \sim y$ if $\sigma^n(x) = y$ for some $n \in \mathbb{Z}$. Show that $\sim$ defines an equivalence relation.

(c) For the bijection $\sigma : \mathbb{Z} \to \mathbb{Z}$ given by $\sigma(m) = m + 5$, determine the equivalence classes of $\sim$.

Question 2. Permutations. $(6+4+5$ points per part) Consider the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 4 & 9 & 6 & 5 & 2 & 3 & 1 & 7 \end{pmatrix} \in S_9.$$

(a) Factor σ into disjoint cycles. Write σ as a product of transpositions. Compute $\mathrm{ord}(\sigma)$ and $\mathrm{sgn}(\sigma)$.

(b) Determine σ^{-1} (in two-line notation) and factor it into disjoint cycles.

(c) Is $\sigma^{-1} = \tau\sigma\tau^{-1}$ for some $\tau \in S_9$? Please justify your answer.

Question 3. Order of an element. $(3+6+6$ points per part)

(a) Give the definition of the **order** of an element g in a group G.

(b) Show that every non-identity element in the group $G = \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \middle| \ a, b, c \in \mathbb{I}_3 \right\}$ (with matrix multiplication) has order 3.

(c) Show that if G is an abelian group, then the subset $H = \{ a \in G \mid a^2 = \mathbb{1} \}$ is a subgroup of G.

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Don't forget to transfer your final work to the page where the question is posed!

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