Math 4320 Name: ______________________

Prelim Part 2 6 March 2015 Time: 50 minutes

INSTRUCTIONS — PLEASE READ THIS NOW

• You will be graded on all the True/False questions, and on the **two** problems you choose of the three remaining problems. Your score will be out of a possible total of 40 points. **On this cover sheet, please circle** the problem numbers for which you wish us to grade. If you do not circle any problems on this cover sheet, we will grade the first three problems you have started to answer.

• Please carefully write all your final answers on the page they are posed. You should include a complete logical justification, and you answers should be written in grammatically correct mathematical language. There are 2 extra blank pages at the end of the exam, but we **will not read** your work on those pages.

• Write your name on this sheet **right now**.

• Look over your test packet **as soon as the exam begins**. If you find any missing pages or problems please ask for another test booklet.

• You have 50 minutes to complete this exam. You may leave early, but if you finish within the last 10 minutes, please remain in your seat.

• This is a closed book exam. You are **NOT** allowed to use a calculator, cell phone, or any other electronic device (not even as a time keeping device).

• **Academic integrity** is expected of every Cornell University student at all times, whether in the presence or absence of a member of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination. I will not discuss this exam with other students until both sections have taken the exam.

Please sign below to indicate that you have read and agree to these instructions.

____

Signature of Student

OFFICIAL USE ONLY

T/F	______	/ 10	______
1.	______	/ 15	______
2.	______	/ 15	______
3.	______	/ 15	______
Total:	______	/ 40	______

True/False. (2 points per part) Please circle **TRUE** if the statement is always true, or **FALSE** if it fails in at least one example. You do **not** need to justify your answer, and I will not read what you write in the spaces below.

(a) On the set $\mathbb{R}^{>0} = \{a \in \mathbb{R} \mid a > 0\}$, the operation $a \star b = a^b$ is an associative operation.

TRUE	FALSE

(b) The map "complex conjugation" $(\mathbb{C} \setminus \{0\}, \cdot) \to (\mathbb{C} \setminus \{0\}, \cdot)$ sending $a + bi$ to $a - bi$ is an isomorphism.

TRUE	FALSE

(c) Every transposition is an odd permutation.

TRUE	FALSE

(d) The set $G = \{\alpha \in S_7 \mid \alpha(3) = 3\}$ is a subgroup of S_7.

TRUE	FALSE

(e) If G is a cyclic group, and d is a divisor of $|G|$, then G contains an element of order d.

TRUE	FALSE

Please answer **two** of the following three questions. The questions are repeated, with space for your answers, on the following pages. **On the cover sheet,** please circle the problems you would like us to grade. If you do not circle any problems on the cover sheet, we will grade the first two problems you have started to answer.

Question 1. Centralizers. ($5 + 5 + 5$ points per part) Let G be a group and $a \in G$. The subset

$$C(a) = \{x \in G \mid xa = ax\}$$

of all elements which commute with a is called the **centralizer** of a.

(a) Show that $C(a)$ is a subgroup of G.

(b) Show that the center of G satisfies $Z(G) = \bigcap_{a \in G} C(a)$.

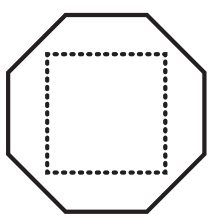
(c) Compute $C((1,2))$ for $(1,2) \in S_5$.

Question 2. Normal subgroups. ($3 + 5 + 7$ points per part)

(a) Give the definition of a **normal subgroup** of a group G.

(b) Show that if $H \leq G$ and $K \lhd G$, then $H \cap K \lhd H$.

(c) Let π_4 be the square and π_8 the regular octagon, both centered at the origin, as in the figure below. Show that $\Sigma(\pi_4)$ is a subgroup of $\Sigma(\pi_8)$. Is it a normal subgroup? Why or why not?



Question 3. Isomorphism. ($3 + 6 + 6$ points per part)

(a) Give the definition of an **isomorphism** of groups.

(b) For a group G and an element $g \in G$, show that conjugation by g (the map $\gamma_g : G \to G$ that sends $\gamma_g(x) = gxg^{-1}$) is an isomorphism.

(c) Which of the following groups are isomorphic? Justify your answers.

- $U(\mathbb{I}_7)$
- $U(\mathbb{I}_{10})$
- $U(\mathbb{I}_{12})$
- $U(\mathbb{I}_{14})$

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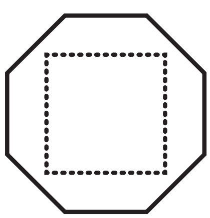

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Don't forget to transfer your final work to the page where the question is posed!

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