

Plenary abstracts:

- **George Boxer, Harvard University:**

Generalized Hasse Invariants for Ekedahl-Oort Strata

The Ekedahl-Oort stratification is the stratification of the moduli space of principally polarized abelian varieties in characteristic p according to the p torsion $A[p]$ of the abelian variety. We will explain how to construct a section of a line bundle on each closed stratum, whose non vanishing locus is precisely the open stratum. For the ordinary locus our construction recovers the classical Hasse invariant. If there is time I will discuss an application to the construction of congruences between automorphic forms.

- **Chantal David, Concordia University:**

Averages of Euler products and statistics of elliptic curves

We present in this talk several results related to statistics of elliptic curves over a finite field $\mathbb{F}_p$, which follow from a general theorem about averages of Euler products. In this general framework, we can reprove known results as the vertical Lang-Trotter conjecture, the vertical Koblitz conjecture, and the vertical Sato-Tate conjecture (for very short intervals). We can also compute statistics for new questions as the problem of amicable pairs and aliquot cycles, first introduced by Silverman and Stange. Our technique is broad and general, and easily applicable to other distribution questions. The starting point of our results is a theorem of Gekeler which gives a reinterpretation of Deuring's theorem in terms of random matrix theory, making a direct connection between the (conjectural) horizontal distributions and the vertical distributions. A key step of our results is to control the stabilization of the Euler factors appearing in Gekeler's theorem: for each ℓ , the Euler factor related to random matrix theory is defined as the r -limit of a matrix count modulo ℓ^r , which stabilizes for some r depending on ℓ and p . Our main result shows that under certain conditions for this stabilization, a weighted average of Euler products is asymptotic to the Euler product of the average factors (which are stable for $r = 1$).

Joint work with D. Koukoulopoulos and E. Smith.

- **Dorian Goldfeld, Columbia University:**

A standard zero free region for Rankin-Selberg L-functions on $GL(n)$

A standard zero-free region for an L-function is a region

$$\operatorname{Re}(s) > 1 - \log(\operatorname{Im}(s))^{-k}$$

for some fixed $k > 0$. Standard zero free regions had not been known previously for Rankin-Selberg L-functions except in special cases. This is joint work with Xiaoqing Li.

- **Chris Hall, University of Wyoming:**

Expander graphs and gonality of curves

We will briefly explain how results on expander graphs can be used to show that certain families of curves have growing gonality. We will also explain the diophantine implications and, if time permits, give an application.

- **Jennifer Park, McGill University:**

Quadratic points on hyperelliptic curves

Using the ideas of Poonen and Stoll, we develop a modified version of Chabauty's method, which shows that a positive proportion of hyperelliptic curves have as few quadratic points as possible.

- **Kartik Prasanna, University of Michigan:**

Extensions of the Gross-Zagier formula

I will first give an introduction to the general conjectural picture relating algebraic cycles to L -functions and discuss some extensions of the Gross-Zagier formula involving p -adic L -functions. This leads naturally to the question of constructing algebraic cycles corresponding to the vanishing of Rankin-Selberg L -functions at the center of symmetry. I will also outline some new constructions of such cycles, based on work in progress with A. Ichino.

- **Shrenik Shah, Columbia University:**

The Spin L -function on GSp_6 via a non-unique model

We give a global integral unfolding to a non-unique model on GSp_6 that represents the partial Spin L -function for a wide class of cuspidal automorphic representations. We deduce that this partial L -function is holomorphic except for a possible simple pole at $s = 1$, and that the presence of such a pole detects images under the exceptional theta lifting from G_2 to $PGSp_6$. This work, which is joint with Aaron Pollack, extends and utilizes previous results of Gan and Gurevich. (<http://arxiv.org/abs/1503.08197>)

- **Xiaoheng Wang, Princeton University:**

Pencils of quadrics and the arithmetic of hyperelliptic curves

In recent joint works with Manjul Bhargava and Benedict Gross, we showed that a positive proportion of hyperelliptic curves over $\mathbb{Q}$ of genus g have no points over any odd degree extension of $\mathbb{Q}$. This is done by computing certain 2-Selmer averages and applying a result of Dokchitser-Dokchitser on the parity of the rank of the 2-Selmer groups in biquadratic twists. In this talk, we will see how arithmetic invariant theory and the geometric theory of pencils of quadrics are used to obtain the 2-Selmer averages.

Contributed talk abstracts:

- **Max Alekseyev, George Washington University:**

On efficient computation of the (number or sum of) multiplicative function inverses

We propose a generic algorithm for computing the inverses of a multiplicative function under the assumption that the set of inverses is finite. We illustrate our algorithm with Eulers totient function and the sum of k -th powers of divisors. Our approach can be further adapted for computing certain functions of the inverses, such as their quantity, sum, or the smallest/largest inverse, which may be computed without and possibly faster than the whole set of inverses. (<http://arxiv.org/abs/1401.6054>)

- **Andrew Bridy, University of Rochester:**

The Artin-Mazur Zeta Function of a Rational Map in Positive Characteristic

The Artin-Mazur zeta function of a dynamical system is a generating function that captures information about its periodic points. In characteristic zero, the zeta function of a rational self-map of $\mathbb{P}^1$ is known to be a rational function. In positive characteristic, the situation is much less clear. I show that the zeta function can be understood for a family of maps in positive characteristic that come from endomorphisms of algebraic groups. Somewhat surprisingly, it typically fails to be rational, and in fact is transcendental.

- **Abel Castillo, University of Illinois at Chicago:**

Bounded gaps between primes in number fields and function fields

In 2013, Zhang, Maynard and Tao proved that there are infinitely many primes that differ by a constant. In this talk we discuss an extension of the Maynard-Tao method to number fields and to the function field $\mathbb{F}_q(T)$.

- **William Chen, Pennsylvania State University:**

Moduli Interpretations for Noncongruence Modular Curves

Quotients of the upper half plane by congruence subgroups of $SL(2, \mathbb{Z})$ are well-known to be moduli spaces parametrizing elliptic curves with an “abelian” level structure. In my talk I will generalize the classical congruence level structures and show that quotients of the upper half plane by noncongruence subgroups are also very naturally moduli spaces for elliptic curves equipped with “nonabelian” level structures.

- **Jaehyun Cho, SUNY Buffalo:**

The average of the smallest prime in a conjugacy class

Let C be a conjugacy class of S_n and K an S_n -fields. Define $n_{K,C}$ to be the smallest prime which is ramified or whose Frobenius automorphism Frob_p does not belong to C . Under some technical conjectures, we compute the average of $n_{K,C}$. For S_3 and S_4 -fields, our result is unconditional. For S_4 and S_5 -fields, we give a different proof which depends on the strong Artin conjecture. Let $N_{K,C}$ to be the smallest prime for which Frob_p belongs to C . For S_3 and S_4 -fields, we obtain an unconditional result for the average of $N_{K,C'}$ where C' is the union of conjugacy classes not in A_n . This is a joint work with Henry Kim.

- **Harris Daniels, Amherst College:**

An Infinite Family of Serre Curves

Given an elliptic curve $E/\mathbb{Q}$, the torsion points of E give rise to a natural Galois representation $\rho_E : \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \rightarrow \text{GL}_2(\widehat{\mathbb{Z}})$ associated to E . In 1972, Serre showed that $[\text{GL}_2(\widehat{\mathbb{Z}}) : \text{Im } \rho_E] \geq 2$ for all non-CM elliptic curves. The main goal of this talk is to exhibit an elliptic

surface such that the Galois representations associated to almost all of the rational specializations have maximal image. Further, we find an explicit set $S \subset \mathbb{Q}$, such that if $t \notin S$, then the Galois representation associated to the specialization at t has maximal image, with a bounded number of exceptions.

- **John Doyle, University of Rochester:**

Preperiodic points for quadratic polynomials over quadratic fields

Given a quadratic polynomial $f(z)$ defined over a number field K , the set of K -rational preperiodic points for $f(z)$ comes naturally equipped with the structure of a (finite!) directed graph $G(f, K)$. A conjecture of Morton and Silverman suggests that, for a fixed positive integer n , there are only finitely many directed graphs that may be realized as $G(f, K)$ for some degree- n number field K and some quadratic polynomial $f(z)$ in $K[z]$. Poonen has given a conditional classification in the $n = 1$ case; that is, in the case where $K = \mathbb{Q}$. I will describe joint work with Xander Faber and David Krumm toward a similar classification over all quadratic extensions of $\mathbb{Q}$.

- **Evan Dummit, University of Rochester:**

Counting Number Fields by Discriminant

The problem of analyzing the number of number field extensions L/K with bounded (relative) discriminant has been the subject of renewed interest in recent years, with significant advances made by Ellenberg-Venkatesh, Kable-Yukie, and (especially) Bhargava. I will give a brief overview of the history of this problem and mention some known (and conjectured) results, and then discuss my work on a series of generalizations, using similar techniques to Ellenberg-Venkatesh, for giving an upper bound on the number of extensions L/K with fixed degree, bounded relative discriminant, and specified Galois closure.

- **Ayla Gafni, Pennsylvania State University:**

Power Partitions

In 1918, Hardy and Ramanujan published a seminal paper which included an asymptotic formula for the partition function. In their paper, they also state without proof an asymptotic equivalence for the number of partitions of a number into k -th powers. In 1934, E. Maitland Wright [Acta Mathematica, 63 (1934) 143–191] gives a very precise asymptotic formula for this restricted partition function, but his argument is quite long and difficult. In this talk, I will present an asymptotic formula for the number of partitions into k -th powers using a relatively simple method, while maintaining a decent error term.

- **Joseph Hundley, SUNY Buffalo:**

A Multi-variable Rankin-Selberg Integral for a Product of $GL(2)$ -twisted Spinor L -functions

This talk is based on joint work with Xin Shen. We consider a new integral representation for $L(s_1, \pi \times \tau_1)L(s_2, \pi \times \tau_2)$, where π is a globally generic cuspidal representation of $GS p_4$, and τ_1 and τ_2 are two cuspidal representations of GL_2 having the same central character. As an application, we find an intriguing connection between the similitude theta correspondence and a certain Fourier coefficient of a residual representation. Time permitting, we discuss a partial extension to $GS p_6$ and the prospect of extending further.

- **Jamie Juul, University of Rochester:**

Galois groups of iterated rational maps and their applications

Given a rational map defined over a field, one can consider the Galois groups of the field extensions generated by adjoining the pre-images of an algebraic point under iterates of

the map. The study of these Galois groups has seen a recent increase in interest due to its many interesting applications. The Chebotarev density theorem allows us to translate statements about the densities of certain significant sets in number theory and dynamics to statements about these Galois groups. In this talk we will discuss some cases in which the structure of these Galois groups are known as well as some of the applications of these results.

- **Cihan Karabulut, CUNY Graduate Center:**

Sums of Binary Hermitian Forms

In one of his papers, Zagier defined a family of functions using quadratic forms. He showed that these functions have very interesting properties and are related to modular forms of integral weight and half integral weight, certain values of Dedekind zeta functions, Diophantine approximation, and continued fractions. He used the theory of periods of modular forms to explain the behavior of these functions. We study a similar family of functions, defining them using binary Hermitian forms. Our preliminary results suggest that this family of functions also has properties similar to Zagier's.

- **Malcolm Kotok, University of Rochester:**

Computing zeta functions of nondegenerate hypersurfaces over finite fields

Zeta functions of varieties over finite fields are generating functions that capture the number of vanishing points of a finite set of polynomial equations. They can be calculated by exhaustively checking for zeros in each finite field; however, this quickly becomes impractical. Using the cohomology theory of Dwork; Sperber and Voight present a deterministic algorithm that is particularly well-suited to work with polynomials in small characteristic that have few monomials (relative to their dimension). I will discuss the algorithm briefly and present my implementation of it written in SAGE.

- **Joseph Kramer-Miller, CUNY Graduate Center:**

p -adic L -functions and the Geometry of the Eigencurve

A major theme in the theory of p -adic deformations of automorphic forms is how p -adic L -functions over eigenvarieties relate to the geometry of these eigenvarieties. In this talk we explain results in this vein for the ordinary part of the eigencurve (i.e. Hida families). We address how Taylor expansions of one variable p -adic L -functions varying over families can detect "bad" geometric phenomena: crossing components of a certain intersection multiplicity and ramification over the weight space. Our methods involve proving a converse to a result of Vatsal relating congruences between eigenforms to their algebraic special L -values and then p -adically interpolating congruences using formal models. These methods should extend to the entire eigencurve provided the existence of an appropriate formal model.

- **Anna Medvedovsky, Brandeis University:**

Lower bounds on dimensions of mod- p Hecke algebras: the nilpotence method

In 2012, Serre and Nicolas revived interest in the study of mod- p Hecke algebras when they proved that for $p = 2$ the Hecke algebra is a power series ring in $\mathbb{F}_2$ generated by T_3 and T_5 . Their methods do not generalize to other primes, but their elementary tools — the Hecke recursion, the nilpotence filtration — serve as the backbone of a new method-in-progress, uniform and elementary, for understanding the structure of mod- p Hecke algebras. I will present this method and compare it with both Nicolas-Serre and Bellaïche-Khare, which

uses deep characteristic-zero theorems to deduce similar results.

- **Stefano Morra, University of Toronto:**

On Serre-type conjectures for 3-dimensional Galois representations

The aim of this talk is to give an overview of recent results on Serre type conjectures for non-semisimple Galois representations of dimension 3. We will show how to relate these problems to potentially semistable local deformation rings and prove modularity of ordinary weights.

- **Filip Najman, University of Zagreb and MIT:**

Torsion of elliptic curves over number fields

We will give an overview of known results about the torsion of elliptic curves over number fields, focusing on recent developments in the subject.

- **Wayne Peng, University of Rochester:**

Wall's Conjecture, ABC Conjecture and Trace sequence

Fibonacci sequence modulo a prime p forms period. Wall's conjecture says the period of modulo p^2 is p times the period of modulo p . We will show that if *abc* conjecture is true, then there are infinite many prime p satisfies Wall's conjecture.

- **Qibin Shen, University of Rochester:**

Function Fields of Class Number One

In 1975, Leitzel, Madan and Queen listed 7 function fields over finite fields (up to isomorphism) with positive genus and class number one and claimed to prove that there are only 7 different function fields. Recently Claudio Stirpe found an 8th one. In our research, we fixed the argument in LMQ to show the 8th one could have been found by their method and is the only one, so the list is now complete.

- **Kevin Vissuet, University of Illinois at Chicago:**

Most subsets are balanced in finite groups

The sumset is one of the most basic and central objects in additive number theory. Many of the most important problems (such as Goldbach's conjecture and Fermat's Last theorem) can be formulated in terms of the sumset $S + S = \{x + y : x, y \in S\}$ of a set of integers S . A finite set of integers A is sum-dominated if $|A + A| > |A - A|$. Though it was believed that the percentage of subsets of $\{0, \dots, n\}$ that are sum-dominated tends to zero, in 2006 Martin and O'Bryant proved a very small positive percentage are sum-dominated if the sets are chosen uniformly at random (through work of Zhao we know this percentage is approximately $4.5 \cdot 10^{-4}$). While most sets are difference-dominated in the integer case, this is not the case when we take subsets of many finite groups. We show that if we take subsets of larger and larger finite groups uniformly at random, then not only does the probability of a set being sum-dominated tend to zero but the probability that $|A + A| = |A - A|$ tends to one, and hence a typical set is balanced in this case.

- **Carl Wang Erickson, Brandeis University:**

Pseudo-modularity and Iwasawa theory

We prove, assuming Greenberg's conjecture, a conjecture of Wake that the ordinary eigen-curve is Gorenstein at an intersection point between the Eisenstein locus and a cuspidal component. As a corollary, we obtain new results on Sharifi's conjecture. This result is

proved by constructing an ordinary pseudodeformation ring and comparing it with the local ring at the intersection point on the eigencurve. In fact, we show that this comparison map is an isomorphism. This is joint work with Preston Wake.

- **Jeffrey Yelton, Pennsylvania State University:**

Galois representations associated to hyperelliptic Jacobians

Let k be a field of characteristic different from 2; let $a_1, \dots, a_d$ be independent transcendental variables over k ; and let J be the Jacobian of the “generic” hyperelliptic curve whose Weierstrass roots are the a_i ’s. I will first describe how investigation of a particular topological monodromy action is used to determine the image of the natural Galois action on the ℓ -adic Tate modules of J , for each prime ℓ . I will explain how these methods may be further used to explicitly describe fields of definition of some torsion subgroups of J .

- **Tian An Wong, CUNY Graduate Center:**

A trace formula condition for an average GRH

According to Weil, the (generalized) Riemann hypothesis is equivalent to the positivity of a certain distribution related to sums over zeroes of the relevant L -function. On the other hand, the Selberg’s trace formula contains families of such distributions arising from Eisenstein series. This relationship gives a condition for a GRH for certain families of L -functions in the context of the formula. In this talk, I will outline the above and discuss further work in progress.

- **Peng Zhao, Yale University:**

Non-gaussian Distribution in Higher Moments of Matrix Elements

We study the distribution in higher moments of matrix elements for the modular surface. Our approach is via Watson’s triple product formula, Kuznetsov trace formula and Rudnick-Soundararajan’s technique to obtain a lower bound of higher moments of L -functions. It turns out that the higher moments of matrix elements does not follow the gaussian distribution.