Self-Diagnostic Review MATH 1110

This collection of questions is a form of self review. It is not important that you get every question correct, nor that you do questions you think are easy. Also some of the questions (particularly the later ones) are quite tricky. If you struggle with large portions of this however it is important to speak to your TA. Finally note this is entirely optional, and will not be graded.

Geometry

1. Find the equation of the line which passes through $(3, -5)$ and has slope -2.

   Using $y = mx + c$, we have that $m = -2$ and substituting in the given point tells us $-5 = -2 \cdot 3 + c$ and so $c = 1$, giving the equation $y = 1 - 2x$.

2. Find the equation of the line which passes through $(2, 3)$ and $(4, 9)$.

   Again using $y = mx + c$ we have $3 = 2m + c$ and $9 = 4m + c$, so multiplying (both sides of) the first equation by two and subtracting the second from this gives $-3 = c$, subbing this into the second gives $6 = 2m$ and so $m = 3$ meaning the solution is $y = 3x - 3$.

3. Find the equation of the line parallel to $y = -2x + 1$ which goes through the point $(10, 10)$.

   We know the slope of the line we want is -2 and we also know $10 = 10 \cdot -2 + c$ and so $c = 30$, hence $y = 30 - 2x$ is the equation.

4. Find the equation of the line perpendicular to $y = \frac{x}{2} - 6$ which goes through the origin.

   If m is the slope of the line we want, we know $m \cdot \frac{x}{2} = -1$ and so $m = -2$. As this line goes through the origin, the c we want is 0 and so $y = -2x$ is what we want.

5. Find the equation of the circle which has as its center the midpoint between $(4, -1)$ and $(2, 5)$ and radius 9.

   Firstly we find the center of the circle. The mid-point between the two given points can be found as the componant-wise average, i.e. $(\frac{4+2}{2}, \frac{-1+5}{2}) = (3, 2)$. Now we know that the equation for the circle has the form $(x-3)^2 + (y-2)^2 = r^2$ where r is the radius. But this is given to be 9 and so $(x-3)^2 + (y-2)^2 = 81$.

Roots, Radicals and Polynomials

6. Simplify the expression $\frac{6x}{\sqrt{x}} + \frac{2x^2}{x^{3/2}}$.

   Using the laws of exponents we have $\frac{6x}{\sqrt{x}} + \frac{2x^2}{x^{3/2}} = 6\sqrt{x} + 2\sqrt{x} = 8\sqrt{x}$.

7. Solve the equation $3\sqrt{x} = x - 4$.

   Squaring both sides gives $9x = x^2 - 8x + 16$ which we rearrange to $0 = x^2 - 17x + 16 = (x-16)(x-1)$. So the possible solutions this are 1 and 16. Testing 1 gives $3 = -3$ which doesn't work (note it would work if we were allowed to use the negative square root), which leaves $3 \cdot 4 = 16 - 4$ which is true. So the solution is 16.

8. Solve the equation $x = 4\sqrt[3]{x}$.

   Cubing both sides gives $x^3 = 64x$ which we rearrange to $0 = x^3 - 64x = x(x^2 - 64) = x(x-8)(x+8)$, so the possible solutions are $0, 8, -8$. By inspection all of these work and so our solutions are $0, 8, -8$.

9. Solve the equation $\sqrt{x+2} + \sqrt{x-2} = \sqrt{4x-2}$.

   Squaring both sides gives $x+2+x-2+2\sqrt{x+2}\sqrt{x-2} = 4x-2$, rearranging gives $x - 1 = \sqrt{(x+2)(x-2)}$ and squaring both sides of this gives $x^2 - 2x + 1 = x^2 - 4$ and thus $2x = 5$ showing $x = 5/2$.

10. Find the maximum value of the function $f(x) = -x^2 - 5x + 9$.

    Completing the square shows $f(x) = -(x^2+5x-9) = -((x+\frac{5}{2})^2 - 9 - \frac{25}{4}) = -((x+\frac{5}{2})^2 - \frac{61}{4})$. As $(x+\frac{5}{2})^2$ is non-negative it is minimized (and so its negative is maximized) at $x = -\frac{5}{2}$ and then it is 0, thus the maximum is given at this point and it equals the hanging constant term $\frac{61}{4}$.

11. Find a quadratic polynomial which has 5 and 1 as the x-intercepts and a minimum value of -12.

    Since 5 and 1 are the x-intercepts the polynomial looks like $k(x-5)(x-1)$ for some constant k. Since the graph of a quadratic is symmetric around the vertical line passing through its vertex, the x coordinate of the vertex is the average of 5 and 1, namely, 3. We know the corresponding y coordinate is -12 and so $-12 = k(3-5)(3-1) = -4k$ and so $k = 3$ giving $f(x) = 3x^2 - 18x + 15$.

Logarithms and Exponentials

12. Find all solutions of $x^{2/3} = 4$.

    Cubing both sides gives $x^2 = 64$ and so $x = \pm 8$.

13. Re-write $2\ln(3x-4) - 5\ln(2x-7)$ as an expression involving a single logarithm.

    We have $2\ln(3x-4) - 5\ln(2x-7) = \ln((3x-4)^2) - \ln((2x-7)^5) = \ln(\frac{(3x-4)^2}{(2x-7)^5})$.

14. Simplify $\frac{x^3(x^4)^5}{x^7(x^2)^4}$.

    $\frac{x^3(x^4)^5}{x^7(x^2)^4} = \frac{x^23}{x^{15}} = x^8$.

15. Simplify $\log_9(3)\log_5(1/25)$.

    $\log_9(3)\log_5(1/25) = \log_9(9^{1/2})\log_5(5^{-2}) = -\log_9(9)\log_5(5) = -1$.

16. Simplify $\ln(\ln(e)) + \log_2(8)$.

    $\ln(\ln(e)) + \log_2(8) = \ln(1) + 3\log_2(2) = 0 + 3 = 3$.

Trigonometry

17. Find $\sin(11\pi/6), \cos(4\pi/3)$ and $\tan(3\pi/4)$.

    $11\pi/6 = 2\pi - \pi/6$, as sine is negative in the 4th quadrant $\sin(11\pi/6) = -\sin\pi/6 = -1/2$. $4\pi/3 = \pi + \pi/3$, Since cosine is negative in the 3rd quadrant, $\cos 4\pi/3 = -\cos\pi/3 = -1/2$. $3\pi/4 = \pi - \pi/4$ and as tangent is negative in the 2nd quadrant, $\tan 3\pi/4 = -\tan\pi/4 = -1$.

18. Find $\tan\theta$ if $\sin\theta = 24/25$ and θ is in the second quadrant.

    $\cos^2\theta = 1 - \sin^2\theta = \frac{49}{625}$, so $\cos\theta = -\frac{7}{25}$ (since cos is negative in 2nd quadrant). Thus $\tan\theta = \frac{\sin\theta}{\cos\theta} = -24/7$.

19. Simplify $(\sin\theta + \cos\theta)^2$.

    Multiplying out gives $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1 + \sin 2\theta$.

20. Simplify $\frac{1}{2}(\sin(a+b) + \sin(a-b))$.

    Using the addition and subtraction formulas for sin we have $\frac{1}{2}(\sin(a+b) + \sin(a-b)) = \frac{1}{2}((\sin a\cos b + \cos a\sin b) + (\sin a\cos b - \cos a\sin b)) = \sin a\cos b$

21. Find all solutions to the equation $\sin\theta = \cos\theta$.

    Dividing both sides by $\cos\theta$ gives $\tan\theta = 1$, since $\tan\pi/4 = 1$ and tangent has period π, $\theta = \pi/4 + n\pi$ for any integer n.

Inequalities

22. Solve the inequality $\frac{x}{2} - 1 < 3x + 9$.

    Multiplying both sides by 2 and rearranging gives $-20 < 5x$ and so $-4 < x$ so $(-4, \infty)$ is the solution.

23. Solve the inequality $|2x - 5| \leq 11$.

    $|2x - 5| \leq 11$ iff $-11 \leq 2x - 5 \leq 11$ iff $-6 \leq 2x \leq 16$ iff $-3 \leq x \leq 8$ and so the solution is $[-3, 8]$.

24. Solve the inequality $3\sqrt{x} - 1 < 5$.

    $3\sqrt{x} - 1 < 5$ iff $3\sqrt{x} < 6$ iff $\sqrt{x} < 2$ iff $x < 4$ and $x \geq 0$ so $[0, 4)$.

25. Solve the inequality $|x^2 - 10| \leq 6$.

    $|x^2 - 10| \leq 6$ iff $-6 \leq x^2 - 10 \leq 6$ iff $4 \leq x^2 \leq 16$ iff $2 \leq \sqrt{x^2} \leq 4$ iff $2 \leq |x| \leq 4$ i.e. $2 \leq x \leq 4$ or $-2 \geq x \geq -4$ and so $[-4, -2] \cup [2, 4]$.