1.1 Lines

22. The line goes through \((-2, \frac{3}{4})\) and \(\left(\frac{3}{5}, \frac{5}{2}\right)\).

\[
m = \frac{\frac{5}{2} - \frac{3}{4}}{\frac{3}{5} - (-2)} = \frac{\frac{10}{4} - \frac{3}{4}}{\frac{3}{5} + \frac{6}{3}} = \frac{\frac{7}{4}}{\frac{8}{3}} = \frac{21}{32}
\]

\[
y - \frac{3}{4} = \frac{21}{32}(x - (-2))
\]

\[
y - \frac{3}{4} = \frac{21}{32}x + \frac{42}{32}
\]

\[
y = \frac{21}{32}x + \frac{42 + 3}{32} = \frac{21}{32}x + \frac{45}{32}
\]

\[
y = \frac{21}{32}x + \frac{33}{32} + \frac{16}{32}
\]

\[
y = \frac{21}{32}x + \frac{55}{32}
\]

36. (a) Write the given line in slope-intercept form.

\[
2x + 3y = 6
\]

\[
y = -\frac{2}{3}x + 2
\]

This line has a slope of \(-\frac{2}{3}\). The desired line has a slope of \(-\frac{5}{3}\) since it is parallel to the given line. Use the definition of slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-1)}{3 - k - 4} = \frac{3}{-1} = \frac{3}{k - 4}
\]

\[
-2(k - 4) = (3)(3)
\]

\[
-2k + 8 = 9
\]

\[
-2k = 1
\]

\[
k = -\frac{1}{2}
\]

(b) Write the given line in slope-intercept form.

\[
5x - 2y = -1
\]

\[
y = \frac{5}{2}x + \frac{1}{2}
\]

This line has a slope of \(\frac{5}{2}\). The desired line has a slope of \(-\frac{2}{5}\) since it is perpendicular to the given line. Use the definition of slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-1)}{3 - k - 4} = \frac{3}{-1} = \frac{3}{k - 4}
\]

\[
-2(k - 4) = (3)(5)
\]

\[
-2k + 8 = 15
\]

\[
-2k = 7
\]

\[
k = -\frac{7}{2}
\]

64. (a) Let \(x\) = age.

\[
u = .85(220 - x) = 187 - .85x
\]

\[
l = .7(220 - x) = 154 - .7x
\]

(b) \(u = 187 - .85(20) = 170\)

\(l = 154 - .7(20) = 140\)

The target heart rate zone is 140 to 170 beats per minute.

(c) \(u = 187 - .85(40) = 153\)

\(l = 154 - .7(40) = 126\)

The target heart rate zone is 126 to 153 beats per minute.

(d) \(154 - .7x = 187 - .85(x + 36)\)

\(154 - .7x = 187 - .85x - 30.6\)

\(154 - .7x = 156.4 - .85x\)

\(.15x = 2.4\)

\(x = 16\)

The younger woman is 16; the older woman is 16 + 36 = 52. \(l = .7(220 - 16) \approx 143\) beats per minute.
1.2 Applications

12. Fixed cost, $100; 50 items cost $1000 to produce.

Let \( C(x) = \) cost of producing \( x \) items.
\[
C(x) = mx + b, \text{ where } b \text{ is the fixed cost.}
\]
\[
C(x) = mx + 100
\]

Now,
\[
C(x) = 1600 \text{ when } x = 50, \text{ so}
\]
\[
1600 = m(50) + 100
\]
\[
1500 = 50m
\]
\[
30 = m.
\]
Thus, \( C(x) = 30x + 100 \).

34. Use the formula derived in Example 7 in this section of the textbook.
\[
F = \frac{9}{5}C + 32
\]
\[
C = \frac{5}{9}(F - 32)
\]
(a) \( C = 37\); find \( F \).
\[
F = \frac{9}{5}(37) + 32
\]
\[
F = \frac{333}{5} + 32
\]
\[
F = 98.6
\]
The Fahrenheit equivalent of 37°C is 98.6°F.

(b) \( C = 36.5\); find \( F \).
\[
F = \frac{9}{5}(36.5) + 32
\]
\[
F = 65.7 + 32
\]
\[
F = 97.7
\]

C = 37.5; find F.
\[
F = \frac{9}{5}(37.5) + 32
\]
\[
F = 67.5 + 32
\]
\[
F = 99.5
\]
The range is between 97.7°F and 99.5°F.

24. (a) Using the points (100, 11.02) and (400, 40.12),
\[
m = \frac{40.12 - 11.02}{400 - 100} = \frac{29.1}{300} = .097.
\]
\[
y - 11.02 = .097(x - 100)
\]
\[
y - 11.02 = .097x - 9.7
\]
\[
y = .097x + 1.32
\]
\[
C(x) = .097x + 1.32
\]
(b) The fixed cost is given by the constant in \( C(x) \). It is $1.32.
(c) \( C(1000) = .097(1000) + 1.32 = 97 + 1.32 = 98.32 \)
The total cost of producing 1000 cups is $98.32.
(d) \( C(1001) = .097(1001) + 1.32 = 97.097 + 1.32 = 98.417 \)
The total cost of producing 1001 cups is $98.417.
(e) Marginal cost = 98.417 - 98.32 = .097 or 9.7¢
(f) The marginal cost for any cup is the slope, .097 or 9.7¢. This means the cost of producing one additional cup of coffee would be 9.7¢.
1.3 Least square fit

<table>
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<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>xy</th>
<th>x²</th>
<th>y²</th>
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<td>12. (a)</td>
<td></td>
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<td>7849.96</td>
<td>400.0</td>
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<td>17.0</td>
<td>1419.5</td>
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<td>14.4</td>
<td>1098.72</td>
<td>5821.69</td>
<td>207.36</td>
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<tr>
<td>1200.6</td>
<td>249.8</td>
<td>20,127.47</td>
<td>96,725.86</td>
<td>4200.56</td>
<td></td>
</tr>
</tbody>
</table>

\[15b + 1200.6m = 249.8\]
\[1200.6b + 96,725.86m = 20,127.47\]

\[b = \frac{249.8 - 1200.6m}{249.8 - 1200.6m}\]

\[b = 249.8 - 1200.6m\]

\[m \approx 0.212\]

\[b = \frac{249.8 - 1200.6 \times 0.212}{15} = -0.315\]

\[Y = 0.212x - 0.315\]

(b) Let \( x = 73 \), find \( Y \).
\[Y = 0.212(73) - 0.315\]
\[\approx 15.2\]

If the temperature were 73°F, you would expect to hear 15.2 chirps per second.

(c) Let \( Y = 18 \), find \( x \).
\[18 = 0.212x - 0.315\]
\[18.315 = 0.212x\]
\[86.4 \approx x\]

When the crickets are chirping 18 times per second, the temperature is 86.4°F.
2.3 Polynomials

12. The graph of \( y = x^4 + 4x^3 - 20 \) has both ends up, at most three turning points, and a y-intercept of \(-20\).
   This is graph F.

40. For a vertical asymptote at \( x = -2 \), put \( x + 2 \) in the denominator. For a horizontal asymptote at \( y = 0 \), the only condition is that the degree of the numerator is less than the degree of the denominator. If the degree of the denominator is \( 1 \), then put a constant in the numerator to make \( y \) approach \( 0 \) as \( x \) gets larger. So, one possible solution is \( y = \frac{3}{x + 2} \).

2.4 Exponential functions

10. The graph of \( y = -2 + 3^{-x} \) is the same as the graph of \( y = 3^{-x} - 2 \). This is the graph of \( y = 3^x \) reflected in the y-axis and translated 2 units downward.
   This is graph B.

16. \( 4^x = 8^{x+1} \)
   \((2^2)^x = (2^3)^{x+1}\)
   \(2^{2x} = 2^{3x+3}\)
   \(2x = 3x + 3\)
   \(-x = 3\)
   \(x = -3\)

30. 4 and 6 cannot be easily written as powers of the same base, so the equation \( 4^x = 6 \) cannot be solved using this approach.
2.5 Logarithm

22. In \( e^2 = x \):
   
   Recall that \( \ln \) means \( \log_e \).
   
   \[
e^x = e^2
   \]
   
   \[
x = 2
   \]

72. If the number \( N \) is proportional to \( m^{-6} \), where \( m \) is the mass, then \( N = km^{-6} \), for some constant of proportionality \( k \).

Taking the common log of both sides, we have

\[
\log N = \log(km^{-6})
\]

\[
= \log k + \log m^{-6}
\]

\[
= \log k - 6 \log m.
\]

This is a linear equation in \( \log m \). Its graph is a straight line with slope \(-6\) and vertical intercept \( \log k \).

76. \( mX + N = m \log_b x + \log_b n \)

\[
= \log_b x^m + \log_b n
\]

\[
= \log_b n x^m
\]

\[
= \log_b y
\]

\[
= Y
\]

Thus, \( Y = mX + N \).

84. \( R(I) = \log \frac{I}{I_0} \)

(a) \( R(1,000,000 \; I_0) \)

\[
= \log \frac{1,000,000 \; I_0}{I_0}
\]

\[
= \log 1,000,000
\]

\[
= 6
\]

(b) \( R(100,000,000 \; I_0) \)

\[
= \log \frac{100,000,000 \; I_0}{I_0}
\]

\[
= \log 100,000,000
\]

\[
= 8
\]

(c) \( R(I) = \log \frac{I}{I_0} \)

\[
6.7 = \log \frac{I}{I_0}
\]

\[
10^{6.7} = \frac{I}{I_0}
\]

\[
I \approx 5,000,000 I_0
\]

(d) \( R(I) = \log \frac{I}{I_0} \)

\[
8.1 = \log \frac{I}{I_0}
\]

\[
10^{8.1} = \frac{I}{I_0}
\]

\[
I \approx 126,000,000 I_0
\]

(e) 1985 quake \( \frac{126,000,000 I_0}{5,000,000 I_0} \approx 25 \)

The 1985 earthquake had an amplitude more than 25 times that of the 1999 earthquake.

(f) \( R(E) = \frac{2}{3} \log \frac{E}{E_0} \)

For the 1999 earthquake,

\[
6.7 = \frac{2}{3} \log \frac{E}{E_0}
\]

\[
10.05 = \log \frac{E}{E_0}
\]

\[
E = 10^{10.05} E_0
\]

For the 1985 earthquake,

\[
8.1 = \frac{2}{3} \log \frac{E}{E_0}
\]

\[
12.15 = \log \frac{E}{E_0}
\]

\[
E = 10^{12.15} E_0
\]

The ratio of their energies is

\[
\frac{10^{12.15} E_0}{10^{10.05} E_0} = 10^{2.1} \approx 126
\]

The 1985 earthquake had an energy about 126 times that of the 1999 earthquake.
(g) Find the energy of a magnitude 6.7 earthquake. Using the formula from part f,
\[
6.7 = \frac{2}{3} \log \frac{E}{E_0}
\]
\[
\log \frac{E}{E_0} = 10.05
\]
\[
\frac{E}{E_0} = 10^{10.05}
\]
\[
E = E_0 10^{10.05}
\]
For an earthquake that releases 15 times this much energy, \(E = E_0 (15) 10^{10.05}\).
\[
R(E_0(15) 10^{10.05}) = \frac{2}{3} \log \left( \frac{E_0(15) 10^{10.05}}{E_0} \right)
\]
\[
= \frac{2}{3} \log(15 \cdot 10^{10.05})
\]
\[
\approx 7.5
\]
So, it's true that a magnitude 7.5 earthquake releases 15 times more energy than one of magnitude 6.7.

2.6 Applications

32. \(f(t) = 500e^{-11t}\)
   
   (a) \(f(t) = 3000\)
   
   \[
   3000 = 500e^{-11t}
   \]
   
   \[
   6 = e^{-11t}
   \]
   
   \[
   \ln 6 = 11t
   \]
   
   \[
   17.9 \approx t
   \]

   It will take 17.9 days.
   
   (b) If \(t = 0\) corresponds to January 1, the date January 17 should be placed on the product. January 18 would be more than 17.9 days.

38. (a) \(A(t) = A_0 \left( \frac{1}{2} \right)^{t/13}\)
   
   \[
   A(100) = 2.0 \left( \frac{1}{2} \right)^{100/13}
   \]
   
   \[
   A(100) \approx .0097
   \]

   After 100 years, about .0097 gram will remain.
   
   (b) \(.1 = 2.0 \left( \frac{1}{2} \right)^{t/13}\)
   
   \[
   \frac{1}{2.0} = \left( \frac{1}{2} \right)^{t/13}
   \]
   
   \[
   \ln .05 = \frac{t}{13} \ln \left( \frac{1}{2} \right)
   \]
   
   \[
   t = \frac{13 \ln .05}{\ln \left( \frac{1}{2} \right)}
   \]
   
   \[
   t \approx 56.19
   \]

   It will take 56 years.