

Remember - We don't expect that everyone will solve every problem, but we do expect that everyone make a serious attempt at every problem and explain what you tried when you can't solve a problem.

Math 1220, Fall 2017

1) Use the partial fraction decomposition

$$\frac{x}{(x-1)(x+1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-2} + \frac{D}{x-3}$$

to find a formula for

$$\int \frac{x}{(x-1)(x+1)(x-2)(x-3)} dx.$$

(Determine the unknown coefficients $A, B, C,$ and D as part of getting your answer.)

2) The substitution $z = \tan \frac{x}{2}$ can be used to convert a rational function of $\sin x$ and $\cos x$ into a rational function of z , which may then be integrated by the method of partial fractions.

a) Find the relationship between dz and dx for this substitution.

b) Starting with the half angle formulas for $\sin \frac{x}{2}$ and $\cos \frac{x}{2}$, show that this transformation gives

$$\begin{aligned}\cos z &= \frac{1-z^2}{1+z^2} \\ \sin z &= \frac{2z}{1+z^2}.\end{aligned}$$

c) Use this technique to calculate

$$\int \frac{1}{\cos x} dx.$$