1. Consider the capital letters of the alphabet, as below, in the sans serif style with no adornments.

\[
\text{ABCDEFGHIJKLMNOPQRSTUVWXYZ} \\
\text{Each letter is a topological space (with the subspace topology inherited from } \mathbb{R}^2). \]

(a) Prove that \( K \) is not homeomorphic to \( X \).

(b) Give an explicit homeomorphism from \( O \) to \( D \). (A picture is fine!!)

(c) Group the letters together into collections of homeomorphic ones.

2. Consider the capital letters of the alphabet, as below, in the sans serif style with no adornments.

\[
\text{ABCDEFGHIJKLMNOPQRSTUVWXYZ} \\
\text{Each letter is a topological space (with the subspace topology inherited from } \mathbb{R}^2). \]

(a) Prove that \( K \) is homotopy equivalent to \( X \). (A sequence of pictures is fine!!)

(b) Show that a Möbius strip and a bagel are each homotopy equivalent to the letter \( O \).

(c) Group the letters together into collections of homotopy equivalent ones.

(d) Which countries’ names, when written in English, consist only of \textbf{contractible} letters? (You might want to check your answer to (b) with the TA to make sure your list of contractible letters is correct.)

4. Take an ordinary strip of white paper. It has two sides. Color one blue and leave the other side white. Give one end of the strip two half twists (also known as a full twist!). Tape the ends together. Do you get a Möbius band? Why or why not? (And if not, what do you get?)

5. Take two strips of paper and put them on top of each other. Twist them together as though you were making a Möbius band, then tape the tops together and the bottoms together. What do you have? What do you get if you cut it lengthwise down the center core if you keep the two bands together? (You may need to do the taping and cutting more than once!)

6. Which of the following surfaces are homeomorphic? Describe with a picture, or explain why not.
7. The following sequence of drawings takes a sphere and deforms it to a torus. Does this sequence describe a homeomorphism? Why or why not?

8. Consider the Klein bottle half filled with apple cider, as in the picture. Describe how you would pour out a glass of cider without cutting open the bottle.

9. Find a circle in the Klein bottle so that if you cut it out, what remains is a Möbius band. (It might help to think about the “quotient” construction of the Klein bottle described in the lecture.)

10. Show that the torus is a 2-fold cover of the Klein bottle. (It might help to think about the “quotient” constructions of the Klein bottle and torus described in the lecture.)