

1. Evaluate the following integrals using integration by parts:

a. $\int \cos^{-1}\left(\frac{x}{2}\right) dx$

b. $\int e^{-2x} \sin 3x dx$

2. Evaluate the integrals. It may be necessary to use a substitution first.

a. $\int \frac{dx}{x(1+\sqrt[3]{x})}$

$$\left[\begin{array}{l} u = \sqrt[3]{x} \\ du = \frac{dx}{3x^{2/3}} \\ dx = 3u^2 du \end{array} \right] \rightarrow \int \frac{3u^2 du}{u^3(1+u)} = 3 \int \frac{du}{u(1+u)} = 3 \ln \left| \frac{u}{u+1} \right| + C = 3 \ln \left| \frac{\sqrt[3]{x}}{1+\sqrt[3]{x}} \right| + C$$

b. $\int \frac{ds}{\sqrt{e^s+1}}$

$$\left[\begin{array}{l} u = \sqrt{e^s+1} \\ du = \frac{e^s ds}{2\sqrt{e^s+1}} \\ ds = \frac{2u du}{u^2-1} \end{array} \right] \rightarrow \int \frac{2u du}{u(u^2-1)} = 2 \int \frac{du}{(u+1)(u-1)} = \int \frac{du}{u-1} - \int \frac{du}{u+1} = \ln \left| \frac{u-1}{u+1} \right| + C = \ln \left| \frac{\sqrt{e^s+1}-1}{\sqrt{e^s+1}+1} \right| + C$$

3. Evaluate the integrals (1) without using a trigonometric substitution, (2) using a trigonometric substitution.

a. $\int \frac{x dx}{\sqrt{4+x^2}}$

(1) $\frac{1}{2} \int \frac{d(4+x^2)}{\sqrt{4+x^2}} = \sqrt{4+x^2} + C$

(2) $[x = 2 \tan y] \rightarrow \int \frac{2 \tan y \cdot 2 \sec^2 y dy}{2 \sec y} = 2 \int \sec y \tan y dy = 2 \sec y + C = \sqrt{4+x^2} + C$

b. $\int \frac{t dt}{\sqrt{4t^2-1}}$

(1) $\frac{1}{8} \int \frac{d(4t^2-1)}{\sqrt{4t^2-1}} = \frac{1}{4} \sqrt{4t^2-1} + C$

(2) $[t = \frac{1}{2} \sec \theta] \rightarrow \int \frac{\frac{1}{2} \sec \theta \tan \theta \cdot \frac{1}{2} \sec \theta d\theta}{\tan \theta} = \frac{1}{4} \int \sec^2 \theta d\theta = \frac{\tan \theta}{4} + C = \frac{\sqrt{4t^2-1}}{4} + C$

4. $\int e^t \sqrt{\tan^2 e^t + 1} dt$