1. *You are planning to use Simpson's Rule to estimate the value of the integral* $\int_1^2 f(x)dx$ *with an error magnitude less than* 10^{-5}. *You have determined that* $|f^{(4)}(x)| \leq 3$ *throughout the interval of integration. How many subintervals should you use to assure the required accuracy? (Remember that for Simpson's Rule the number has to be even.)*

2. *Evaluate the improper integrals.*

   a. $\int_{-\infty}^{0} xe^{3x}dx$

   b. $\int_{-\infty}^{\infty} \frac{4dx}{x^2+16}$

   c. $\int_{-2}^{\infty} \frac{d\theta}{(\theta+1)^{3/5}}$

3. *Which of the improper integrals converge and which diverge?*

   a. $\int_0^\infty e^{-u}\cos u du$

   b. $\int_1^\infty \frac{e^{-t}}{\sqrt{t}}dt$

   c. $\int_{-\infty}^{\infty} \frac{dx}{x^2(1+e^x)}$

   sol) $\int_{-\infty}^{\infty} \frac{dx}{x^2(1+e^x)} = \int_{-\infty}^{-1} \frac{dx}{x^2(1+e^x)} + \int_{-1}^{0} \frac{dx}{x^2(1+e^x)} + \int_0^1 \frac{dx}{x^2(1+e^x)} + \int_1^\infty \frac{dx}{x^2(1+e^x)}$; The reason that we need four subintervals of the integration is 1) the function $\frac{dx}{x^2(1+e^x)}$ is discontinuous at $x = 0$ and so two limits are required for 0^- and 0^+; and 2) the interval of the integrations includes ∞ and $-\infty$, and so two limits are required for ∞ and $-\infty$. Then $\lim_{x\to 0} \frac{\frac{1}{x^2}}{\frac{1}{x^2(1+e^x)}} = \lim_{x\to 0} \frac{x^2(1+e^x)}{x^2} = \lim_{x\to 0}(1+e^x) = \frac{1}{2}$ and $\int_0^1 \frac{dx}{x^2}$ diverges $\Rightarrow \int_0^1 \frac{dx}{x^2(1+e^x)}$ diverges by another version of Limit Comparison Test that I mentioned in section $\Rightarrow \int_{-\infty}^{\infty} \frac{dx}{x^2(1+e^x)}$ diverges.