Definition 0.1 (Infinite Sequences) *An infinite sequence of numbers is a function whose domain is the set of positive integers.*

Ex 1. *Write out the first few terms of the sequences* $a_1 = 1,\ a_{n+1} = a_n + \frac{1}{2^n}$.

Ex 2. *Find a formula for the nth term of the sequence.*

a. *The sequence* $1,\ -4,\ 9,\ -16,\ 25, \ldots$

b. *The sequence* $1,\ 0,\ 1,\ 0,\ 1, \ldots$

Theorem 0.2 *Suppose that* $f(x)$ *is a function defined for all* $x \geq n_0$ *and that* $\{a_n\}$ *is a sequence of real numbers such that* $a_n = f(n)$ *for* $n \geq n_0$. *Then*

$$\lim_{x\to\infty} f(x) = L \Rightarrow \lim_{x\to\infty} a_n = L$$

The significance of Theorem 0.2 is to enable us to use L'Hôpital's Rule to find the limits of some sequences.

Ex 3. *Find* $\lim_{n\to\infty} a_n$ *when* $a_n = \frac{1-5n^4}{n^4+8n^3}$.

Ex 4. *Find* $\lim_{n\to\infty} a_n$ *when* $a_n = \frac{(\ln n)^{200}}{n}$.

Definition 0.3 (Geometric Series) *a. If* $|r| < 1,\ \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$.

b. If $|r| \geq 1,\ \sum_{n=1}^{\infty} ar^{n-1} = \infty$. *Diverges.*

Ex 5. *Find* $\sum_{n=1}^{\infty} \frac{7}{4^n}$.

Ex 6. *Find* $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{(-1)^n}{5^n}\right)$.

Theorem 0.4 *a. If* $\sum_{n=1}^{\infty} a_n$ *converges, then* $a_n \to 0$. *Note that the converse is not true, i.e.* $a_n \to 0 \not\Rightarrow \sum_{n=1}^{\infty} a_n$ *converges. For example,* $a_n = \frac{1}{n}$.

b. If a_n *fails to exist or is difference from zero, then* $\sum_{n=1}^{\infty} a_n$ *diverges.*

Ex 7. *Does* $\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^n$ *converge or diverge? If a series converge, find its sum.*

Ex 8. *Does* $\sum_{n=0}^{\infty} (-1)^{n+1} n$ *converge or diverge? If a series converge, find its sum.*