Theorem 0.1 (The nth Term Test) *If a_n fails to exist or is different from zero, then $\sum_{n=1}^{\infty} a_n$ diverges.*

Theorem 0.2 (The Alternating Series Test) *The series*

$$\sum_{n=1}^{\infty}(-1)^{n+1}u_n = u_1 - u_2 + u_3 - u_4 + \dots \tag{1}$$

converges if all three of the following conditions are satisfied:

a. The u_n's are all positive.

b. $u_n \geq u_{n+1}$ for all $n \geq N$, for some integer N.

c. $u_n \to 0$ as $n \to \infty$.

Definition 0.3 (Absolutely Convergent) *A series $\sum a_n$ converges absolutely if the corresponding series of absolutel values $\sum |a_n|$ converges.*

Definition 0.4 (Conditionally Convergent) *A series that converges but does not converge absolutely converges conditionally.*

Theorem 0.5 (The Absolute Convergence Test) *If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.*

Which of the series in the following converge absolutely, or conditionally, and which diverge?

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$

2. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{\ln n}$

3. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3+n}{5+n}$

4. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\tan^{-1} n}{n^2+1}$

5. $\sum_{n=1}^{\infty}(-1)^{n+1}(\sqrt{n+\sqrt{n}}-\sqrt{n})$

6. *In the series $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n x^n$, (a) find the series' radius and interval of convergence. For what values of x does the series converge (b) absolutely, (c) conditionally?*

Definition 0.6 (Taylor and Maclaurin series) *Let f be a function with derivatives of all orders throughout some interval containing a as an interior point. Then Taylor series generated by f at $x = a$ is*

$$\sum_{k=0}^{\infty}\frac{f^{(k)}(a)}{k!}(x-a)^k \tag{2}$$

The Maclaurin series generated by f is $\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}x^k$, the Taylor series generated by f at $x = 0$.

Definition 0.7 (Taylor Polynomial of order n) *displaystyle*$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

6. *Find the Taylor polynomials of orders* $0,\ 1,\ 2$ *and* 3.

   a. $f(x) = \frac{1}{x},\ a = 2$

   b. $f(x) = \sin x,\ a = \frac{\pi}{4}$

7. Find the Malaurin series for

   a. e^{-x}

   b. $\sin 3x$

8. Find the Taylor series

   a. $f(x) = \frac{1}{x^2},\ a = 1$

   b. $f(x) = \frac{x}{1-x},\ a = 0$