

1. Evaluate the integrals in (a) and (b).

a.  $\int \frac{(2r-1) \cos \sqrt{3(2r-1)^2+6}}{\sqrt{3(2r-1)^2+6}} dr$

b.  $\int \frac{\sin \sqrt{\theta}}{\sqrt{\theta} \cos^3 \sqrt{\theta}} d\theta$

2.  $\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$

3. Find the area of the region enclosed by  $y = x^4 - 4x^2 + 4$  and  $y = x^2$ .

4. Find the volume of the solid generated by revolving the region bounded by the parabola  $y = x^2$  and the line  $y = 1$  about the line  $y = -1$ .

5. Volume of a bowl

a. A hemispherical bowl of radius  $a$  contains water to a depth  $h$ . Find the volume of water in bowl.

$$R(y) = \sqrt{a^2 - y^2} \Rightarrow V = \pi \int_{-a}^{-a+h} (a^2 - y^2) dy = \pi [a^2 y - y^3/3]_{-a}^{-a+h} = \pi [a^2 h - a^3 - (h-a)^3/3 - (-a^3 + a^3/3)] = \pi [a^2 h - 1/3(h^3 - 3h^2 a + 3ha^2 - a^3) - a^3/3] = \pi (a^2 h - h^3/3 + h^2 a - ha^2) = \pi h^2(3a - h)/3$$

b. Water runs into a sunken concrete hemispherical bowl of radius 5m at the rate of 0.2 m<sup>3</sup>/sec. How fast is the water level in the bowl rising when the water is 4m deep?

$$\text{Given } \frac{dV}{dh} = 0.2 \text{ m}^3/\text{sec} \text{ and } a = 5 \text{ m, find } \frac{dh}{dt} \Big|_{h=4}. \text{ From part (a), } V(h) = \pi h^2(15 - h)/3 = 5\pi h^2 - \pi h^3/3 \Rightarrow \frac{dV}{dh} = 10\pi h - \pi h^2 \Rightarrow \frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} = \pi h(10 - h) \frac{dh}{dt} \Rightarrow \frac{dh}{dt} \Big|_{h=4} = \frac{0.2}{4\pi(10-4)} = \frac{1}{(20\pi)(6)} = \frac{1}{120\pi} \text{ m/sec.}$$