Math 191 **Practice Prelim 1** **Fall 2005**

No calculators, notes or books allowed.

To improve the chance for partial credit and also generally ease the work of grading, please:

- write clearly and be well organized;
- use the page backs for ungraded scrap work and for checking your answers;
- box in your answers; and
- reduce your answers as much as possible. None of the calculations are long and none of the answers are long.

1. Evaluate the following expressions:

a) $\int \sec^2(x)dx$

b) $\dfrac{d}{dt}\displaystyle\int_{t^3}^{1} \cos^4(x)dx$

c) $\displaystyle\int_{-2}^{2} \frac{x^3}{1+x^6}\, dx$

2. Consider the region bounded on the left by the y-axis and on the right by the curves $y = \sin(x)$ and $y = \cos(x)$. Find the area of the region.

3. a) Express the area under the curve $y = \sin(x)$ between $x = 0$ and $x = \frac{\pi}{6}$ as a limit of Riemann sums. (Use uniform partitions and the right hand rule.)

b) Find

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^7}{n^8}$$

4. Evaluate

a) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\, dx$

b) $\displaystyle\int x^2\sqrt{2-x}\, dx$

c) $\displaystyle\int_0^1 \sqrt{1-\sin(y)}\sqrt{1-\sin^2(y)}\, dy.$

5. Consider the region in the first quadrant bounded on the left by the y–axis, above by the line $y = 2 - x$, and below by the line $y = x$.

a) Find the volume of the solid generated by revolving this region about the y–axis.

b) Find the volume of the solid when rotated about the x–axis.

Some formulas:

$\cos(a+b) = \cos a\cos b - \sin a\sin b$ $\qquad$ $\sin(a/2) = \pm\sqrt{\frac{1-\cos a}{2}}$

$\sin(a+b) = \cos a\sin b + \sin a\cos b$ $\qquad$ $\cos(a/2) = \pm\sqrt{\frac{1+\cos a}{2}}$