Practice Prelim 2, Math 191, Fall 2005

No calculators. Show your work. Clearly mark each answer.

1. Consider the curve $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$.

   (a) Find the length of this curve between $x = 1$ and $x = e^4$.

   (b) Consider the segment of the curve between $x = 1$ and $x = e$. Verify that this piece of curve lies above the x-axis and find the area of the surface obtained by rotating it around the x-axis.

2. (a) For x in $\left[0, \frac{\pi}{2}\right]$, what is $\sin^{-1}(\cos x)$?

   (b) Evaluate $\cos(\tan^{-1} x)$.

   (c) Determine the equation of the line tangent to the graph of $y = \tan^{-1}(\ln x)$ at $x = e$.

3. A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 2 hours there are 10,000 bacteria. At the end of 5 hours there are 70,000. How many bacteria were present initially? *(Simplify your answer but do not evaluate it numerically.)*

4. (a) Prove or disprove:

   (i) $\tan^{-1} x = O(1)$ (ii) $x^{-2}\,3^x$ grows slower than $x\,2^x$

   (iii) $\log_2 3^{x^2}$ grows at the same rate as $(x+7)^2$ (iv) $\dfrac{1}{x} = o\left(\dfrac{1}{\ln x}\right)$

   (b) If $f = O(g)$ and $g = O(h)$, is it true that $f = O(h)$? Explain.

5. Evaluate the following integrals:

   (a) $\displaystyle\int x\,e^{-x^2}\,dx$ (b) $\displaystyle\int \frac{dx}{\sqrt{x - x^2}}$

   (c) $\displaystyle\int x^a \ln x\,dx \quad (a \neq -1)$ (d) $\displaystyle\int \frac{2x-1}{x^2+2x+2}\,dx$

6. For each integer $n \geq 0$, let $I_n = \displaystyle\int_0^{\pi/4} \tan^n x\,dx$.

   (a) Find I_0 and I_1.

   (b) Find a formula expressing I_{n+2} in terms of I_n.

   (c) Deduce a formula expressing I_{n+4} in terms of I_n. Hence (or otherwise) find I_4 and I_5.