Practice Prelim 2, Math 191, Fall 2006

1. (a) Show that the function $f(x) = x^3 + 3x$ is one-to-one and hence has an inveerse $f^{-1}(x)$.

   (b) What are the domain and the range of $f^{-1}(x)$?

   (c) Find $\dfrac{d}{dx} f^{-1}(x)$ at $x = 4$.

2. Find the global maximum of $f(x) = x^{1/x}$ in the domain $x > 0$. *Hint:* $x^{1/x} = e^{\frac{1}{x} \ln x}$.

3. Let $f(x) = \sin^{-1}(\tan x)$.

   (a) What is the natural domain for $f(x)$ so that the inverse function $f^{-1}(x)$ exists?

   (b) Calculate the inverse function $f^{-1}(x)$ and specify its domain.

4. Suppose the rate at which a rumor spreads - that is, the number of people who have heard the rumor over a period of time - increases with the number of people who have heard it. If y is the number of people who have heard the rumor, then

$$\frac{dy}{dt} = ky,$$

   where t is the time in days.

   (a) If y is 1 when $t = 0$ and y is 5 when $t = 2$, find k.

   (b) Using the value of k from (a), find y when $t = 3$.

5. True or false? Give a reason for each answer.

   a) $e^{\log_2 x} = O(x)$.

   b) $\tanh x = o(\sqrt{x})$.

   c) If $f = o(g)$, $g = o(h)$, then $fg = o(h)$.

6. Evaluate the following.

   a) $\sin(\tan^{-1} x)$.

   b) $\sin(\sec^{-1}(x/6))$.

[28] 7. Evaluate the following:

(a) $\displaystyle\int \frac{x}{\sqrt{-3+4x^2-x^4}}dx$

(b) $\displaystyle\int_0^{\sqrt{2}} \frac{dx}{x^4+3x^2+2}$

(c) $\displaystyle\int \frac{1+\tan^2 x}{1-\tan^2 x}dx$

(d) $\displaystyle\int \frac{dx}{5x\sqrt{\ln(3x)}}dx$