Practice Prelim 3, Math 191, Fall 2005

No calculators. Clearly mark each answer.

1. Decide, giving reasons, whether the following series converges absolutely, converges conditionally, or diverges?

$$a) \quad \sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{3/2}} \qquad [10\,points]$$

$$b) \sum_{n=2}^{\infty} \frac{1}{n + \sin n} \qquad [5\,points]$$

$$c) \sum_{n=2}^{\infty} \frac{n\sqrt{n+1}}{n^3 + 3n + 1} \qquad [5\,points]$$

$$d) \sum_{n=1}^{\infty} \frac{(2n+1)!^2}{(3n)!} \qquad [10\,points]$$

2. a) Find the Maclaurin series for the function

$$f(x) = \frac{x^2}{1+x}$$

For what values of x does the series converge absolutely?

b) Does the series converge at $x = 1$? Explain. [20 points]

3. a) Find the sum of the series

$$1 + \frac{2}{10} + \frac{3}{10^2} + \frac{2}{10^4} + \frac{3}{10^5} + \frac{2}{10^7} + \frac{3}{10^8} + ... \qquad [10\,points]$$

b) Does the series

$$b) \sum_{n=1}^{\infty} \left(\sin \frac{1}{2n} - \sin \frac{1}{2n+1} \right) \qquad [10\,points]$$

converge? Why or why not?

4. Evaluate the integral

$$-\int_0^1 \ln x \, dx \qquad [10\,points]$$

5. According to the error-bound formula for Simpson's rule (the formula would be given on the actual exam), how many sub-intervals should you use to be sure of estimating the value of

$$\ln 3 = \int_1^3 \frac{1}{x}\, dx$$

by Simpson's rule with an error no more than 10^{-2} in absolute value? (Remember that for Simpson's rule, the number of sub-intervals has to be even). [20 points]