

Math 191 Fall 2004 Prelim 1

9/21/04

Print your name, your section number, and your TA's name on the exam booklet. Fill in the date and sign the academic integrity statement. NO CALCULATORS, COMMUNICATION DEVICES, BOOKS, OR NOTES ARE TO BE CONSULTED, with the exception of one $8\frac{1}{2}'' \times 11''$ sheet of notes. Show all your work, but no need to simplify answers.

1 (10 pts) Find the derivatives of the following functions:

(a) $f(x) = \int_0^x \cos t \, dt$ (b) $g(x) = \int_0^{\sin x} t^2 \, dt$

2 (10 pts) Evaluate the following definite integrals:

(a) $\int_0^2 3x^2 \sqrt{x^3 + 1} \, dx$ (b) $\int_0^{\pi/2} \cos x \cos(\sin x) \, dx$

3 (16 pts) Determine the number of subintervals needed to approximate the integral of $f(x) = x^3 + 2x^2 + x - 1$ from $x = 0$ to $x = 2$, using the trapezoidal rule, to within an absolute error of $\frac{1}{100}$.

4 (16 pts)

(a) Suppose a function, $f(x)$, has an average value of 8 on the interval $[0, 1]$ and an average value of 4 on the interval $[1, 2]$. Show that the average value of f on $[0, 2]$ is 6.

(b) If f has an average value of 3 on $[3, 4]$ and an average value of 9 on $[4, 6]$ then what is the average value of f on $[3, 6]$?

5 (16 pts) The base of a solid is the region in the xy -plane enclosed by the two parabolas $y = 2 - x^2$ and $y = -6 + x^2$. If the cross-sections perpendicular to the x -axis are right isosceles triangles with side lying on the xy -plane then what is the volume of the solid?

6 (16 pts) Find the volume of the solid of revolution generated by revolving the region enclosed by $x = 2 - \cos y$, $x = \sin y$, $y = 0$, and $y = \pi$ about the y -axis.

Note: $\cos^2 y - \sin^2 y = \cos(2y)$

7 (16 pts) Find the volume of the solid of revolution generated by revolving the region enclosed by $y = 12(x^2 - x^3)$ and the x -axis in the first quadrant about the y -axis. The region is graphed below.

