Your name: _____

Your TA's name: _____

Your section number and/or day and time: _____

Math 191, Prelim 3

Tuesday November 28, 2006. 7:30 – 9:00 PM

This exam should have 8 pages, with 6 problems adding up to 100 points plus and an extra credit problem worth 5 points.

The last two pages are blank and can be used as scrap paper for computations and checking answers.

No calculators, notes or books allowed.

To improve your chances of getting full credit (or maximum partial credit) and to ease the work of the graders, please:

- write clearly and legibly;
- box in your answers;
- simplify your answers as much as possible;
- explain your answers as completely as time and space allow.

Problem 1:	/18
Problem 2:	/21
Problem 3:	/10
Problem 4:	/10
Problem 5:	/21
Problem 6:	/20
Extra Credit:	/5

TOTAL:	**/100 +5**

1. (6 pts. each) Find the limit if it exists.

   (a) $\lim_{n\to\infty} \dfrac{2n^2+1}{\sqrt{n^4+n^3+1}}$

(b) $\lim_{n\to\infty} (n+1)^{\frac{1}{n}}$

(c) $\lim_{n\to\infty} \sqrt{n+2} - \sqrt{n}$

2. (7 pts. each) Do the following converge absolutely, converge conditionally or diverge? Be careful to explain your reasons and calculations.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2n^2+1}{\sqrt{n^4+n^3+1}}$

(b) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln(n)}$

(c) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{\sin(n)}{n}\right)^3$

3. (10 pts.) Find the sum

$$\sum_{n=0}^{\infty} \int_{n}^{n+1} e^{-x} dx.$$

4. (10 pts.) Write the following as a fraction.

$$.200620062006 \cdots = .\overline{2006}$$

5. (a) (14 pts.) Find the Taylor series of $\sin(x)$ at $x = \pi$.

(b) (7 pts.) Find the radius and interval of convergance for the Taylor series you obtained in part a).

6. (a) (15 pts.) If you wanted to estimate the integral $\int_0^{\pi/2} \sin(2x)$ using the trapezoid rule, how many subintervals would you have to make for your estimate to be accurate to 2 decimal places? (You may leave your answer in an exact form, i.e. without decimals. Only the number of subintervals is requested. You do not need to calculate the estimate of $\int_0^{\pi/2} \sin(2x)$ itself.)
The error for the trapezoid rule is $|E_T| \leq \frac{M(b-a)^3}{12n^2}$, where M is the maximum of the second derivative of the function in the interval $[a, b]$, and n is the number of intervals.

(b) (5 pts.) Is the estimate in part a) an overestimate, and underestimate, or exact. Explain your answer.

Extra credit (5 pts.) : Find the sum

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)2^n}.$$