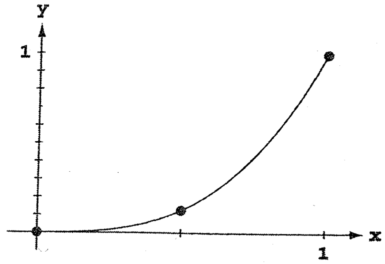
HW #1 Solutions

5.1 Estimating with Finite Sums

2. $f(x) = x^3$

Since f is increasing on [0, 1], we use left endpoints to obtain lower sums and right endpoints to obtain upper sums.



(a) $\triangle x = \frac{1-0}{2} = \frac{1}{2}$ and $x_i = i\triangle x = \frac{i}{2} \Rightarrow$ a lower sum is $\sum_{i=0}^{1}\left(\frac{i}{2}\right)^3 \cdot \frac{1}{2} = \frac{1}{2}\left(0^3 + \left(\frac{1}{2}\right)^3\right) = \frac{1}{16}$

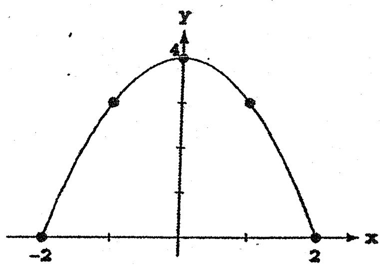
(b) $\triangle x = \frac{1-0}{4} = \frac{1}{4}$ and $x_i = i\triangle x = \frac{i}{4} \Rightarrow$ a lower sum is $\sum_{i=0}^{3}\left(\frac{i}{4}\right)^3 \cdot \frac{1}{4} = \frac{1}{4}\left(0^3 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3\right) = \frac{36}{256} = \frac{9}{64}$

(c) $\triangle x = \frac{1-0}{2} = \frac{1}{2}$ and $x_i = i\triangle x = \frac{i}{2} \Rightarrow$ an upper sum is $\sum_{i=1}^{2}\left(\frac{i}{2}\right)^3 \cdot \frac{1}{2} = \frac{1}{2}\left(\left(\frac{1}{2}\right)^3 + 1^3\right) = \frac{1}{2} \cdot \frac{9}{8} = \frac{9}{16}$

(d) $\triangle x = \frac{1-0}{4} = \frac{1}{4}$ and $x_i = i\triangle x = \frac{i}{4} \Rightarrow$ an upper sum is $\sum_{i=1}^{4}\left(\frac{i}{4}\right)^3 \cdot \frac{1}{4} = \frac{1}{4}\left(\left(\frac{1}{4}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^3 + 1^3\right) = = \frac{100}{256} = \frac{25}{64}$

4. $f(x) = 4 - x^2$

Since f is increasing on [−2, 0] and decreasing on [0, 2], we use left endpoints on [−2, 0] and right endpoints on [0, 2] to obtain lower sums and use right endpoints on [−2, 0] and left endpoints on [0, 2] to obtain upper sums.



(a) $\triangle x = \frac{2-(-2)}{2} = 2$ and $x_i = -2 + i\triangle x = -2 + 2i \Rightarrow$ a lower sum is $2 \cdot \left(4 - (-2)^2\right) + 2 \cdot \left(4 - 2^2\right) = 0$

(b) $\triangle x = \frac{2-(-2)}{4} = 1$ and $x_i = -2 + i\triangle x = -2 + i \Rightarrow$ a lower sum is $\sum_{i=0}^{1}\left(4 - (x_i)^2\right) \cdot 1 + \sum_{i=3}^{4}\left(4 - (x_i)^2\right) \cdot 1$
$= 1\left(\left(4 - (-2)^2\right) + \left(4 - (-1)^2\right) + \left(4 - 1^2\right) + \left(4 - 2^2\right)\right) = 6$

(c) $\triangle x = \frac{2-(-2)}{2} = 2$ and $x_i = -2 + i\triangle x = -2 + 2i \Rightarrow$ a upper sum is $2 \cdot \left(4 - (0)^2\right) + 2 \cdot \left(4 - 0^2\right) = 16$

(d) $\triangle x = \frac{2-(-2)}{4} = 1$ and $x_i = -2 + i\triangle x = -2 + i \Rightarrow$ a upper sum is $\sum_{i=1}^{2}\left(4 - (x_i)^2\right) \cdot 1 + \sum_{i=2}^{3}\left(4 - (x_i)^2\right) \cdot 1$
$= 1\left(\left(4 - (-1)^2\right) + \left(4 - 0^2\right) + \left(4 - 0^2\right) + \left(4 - 1^2\right)\right) = 14$

10. (a) $D \approx (1)(300) + (1.2)(300) + (1.7)(300) + (2.0)(300) + (1.8)(300) + (1.6)(300) + (1.4)(300) + (1.2)(300)$
$+ (1.0)(300) + (1.8)(300) + (1.5)(300) + (1.2)(300) = 5220$ meters (NOTE: 5 minutes = 300 seconds)

(b) $D \approx (1.2)(300) + (1.7)(300) + (2.0)(300) + (1.8)(300) + (1.6)(300) + (1.4)(300) + (1.2)(300) + (1.0)(300)$
$+ (1.8)(300) + (1.5)(300) + (1.2)(300) + (0)(300) = 4920$ meters (NOTE: 5 minutes = 300 seconds)

HW #1 Solutions

MATH 191 (Spring 2008)
Professor: Dennis Yang
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14. (a) The speed is a decreasing function of time $\Rightarrow$ right end-points give an lower estimate for the height (distance) attained. Also
(left) (upper)

t	0	1	2	3	4	5
v	400	368	336	304	272	240

gives the time-velocity table by subtracting the constant $g = 32$ from the speed at each time increment $\Delta t = 1$ sec. Thus, the speed ≈ 240 ft/sec after 5 seconds. An upper estimate = 272 ft/sec

(b) A lower estimate for height attained is $h \approx [368 + 336 + 304 + 272 + 240](1) = 1520$ ft.

Section 5.2 Sigma Notation and Limits of Finite Sums

2. $\displaystyle\sum_{k=1}^{3} \frac{k-1}{k} = \frac{1-1}{1} + \frac{2-1}{2} + \frac{3-1}{3} = 0 + \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$

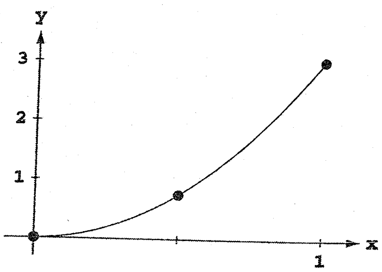
6. $\displaystyle\sum_{k=1}^{4} (-1)^k \cos k\pi = (-1)^1 \cos(1\pi) + (-1)^2 \cos(2\pi) + (-1)^3 \cos(3\pi) + (-1)^4 \cos(4\pi)$

$$= -(-1) + 1 - (-1) + 1 = 4$$

16. $\displaystyle\sum_{k=1}^{5} (-1)^k \frac{k}{5}$

24. $\displaystyle\sum_{k=1}^{6} (k^2 - 5) = \sum_{k=1}^{6} k^2 - \sum_{k=1}^{6} 5 = \frac{6(6+1)(2\cdot 6+1)}{6} - 5\cdot 6 = 61$

38. $f(x) = 3x^2$



Since f is increasing on $[0, 1]$ we use right endpoints to obtain upper sums. $\Delta x = \frac{1-0}{n} = \frac{1}{n}$ and $x_i = i\Delta x = \frac{i}{n}$. ~~So~~ an upper sum is $\displaystyle\sum_{i=1}^{n} 3x_i^2\left(\frac{1}{n}\right) = \sum_{i=1}^{n} 3\left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right) = \frac{3}{n^3}\sum_{i=1}^{n} i^2 = \frac{3}{n^3}\cdot\left(\frac{n(n+1)(2n+1)}{6}\right)$

$\displaystyle = \frac{2n^3 + 3n^2 + n}{2n^3} = \frac{2 + \frac{3}{n} + \frac{1}{n^2}}{2}$. Thus, $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} 3x_i^2\left(\frac{1}{n}\right)$

$\displaystyle = \lim_{n\to\infty}\left(\frac{2 + \frac{3}{n} + \frac{1}{n^2}}{2}\right) = \frac{2}{2} = 1.$