8.2: 4

$$\begin{array}{ccc} & & \sin x \\ x^2 & \xrightarrow{(+)} & -\cos x \\ 2x & \xrightarrow{(-)} & -\sin x \\ 2 & \xrightarrow{(+)} & \cos x \\ 0 & & \end{array} \quad \int t^2\cos t dt = t^2\sin t + 2t\cos t - 2\sin t + C$$

8.2: 8 $u = \sin^{-1} y,\ du = \frac{dy}{\sqrt{1-y^2}}; dv = dy, v = y;\ \int \sin^{-1} y dy = y\sin^{-1} y - \int \frac{y dy}{\sqrt{1-y^2}} = y\sin^{-1} y + \sqrt{1-y^2} + C$

8.2: 24 $\int e^{-2x}\sin 2x dx; [y = 2x] \to \frac{1}{2}\int e^{-y}\sin y dy = I;\ [u = \sin y,\ du = \cos y dy;\ dv = e^{-y}dy,\ v = e^{-y}] \Rightarrow I = \frac{1}{2}\left(-e^{-y}\sin y + \int e^{-y}\cos y dy\right)[u = \cos y,\ du = -\sin y;\ dv = e^{-y}dy, v = -e^{-y}] \Rightarrow I = -\frac{1}{2}e^{-y}\sin y + \frac{1}{2}\left(-e^{-y}\cos y - \int(-e^{-y})(-\sin y)dy\right) = -\frac{1}{2}e^{-y}(\sin y + \cos y) - I + C' \Rightarrow 2I = -\frac{1}{2}e^{-y}(\sin y + \cos y) + C' \Rightarrow I = -\frac{1}{4}e^{-y}(\sin y + \cos y) + C = -\frac{e^{-2x}}{4}(\sin 2x + \cos 2x) + C$, where $C = \frac{C'}{2}$

8.2: 26 $u = x, du = dx;\ dv = \sqrt{1-x}dx,\ v = -\frac{2}{3}\sqrt{(1-x)^3};\ \int_0^1 x\sqrt{1-x}dx = [-\frac{2}{3}\sqrt{(1-x^3)}x]_0^1 + \frac{2}{3}\int_0^1 \sqrt{(1-x)^3}dx = \frac{2}{3}\left[-\frac{2}{5}(1-x)^{5/2}\right]_0^1 = \frac{4}{15}$

8.2: 28 $u = \ln(x+x^2),\ du = \frac{(2x+1)dx}{x+x^2};\ dv = dx, v = x;\ \int \ln(x+x^2)dx = x\ln(x+x^2) - \int \frac{2x+1}{x(x+1)} \cdot x dx = x\ln(x+x^2) - \int \frac{(2x+1)dx}{x+1} = x\ln(x+x^2) - \int \frac{2(x+1)-1}{x+1}dx = x\ln(x+x^2) - 2x + \ln|x+1| + C$

8.3: 8 $\frac{t^4+9}{t^4+9t^2} = 1 + \frac{-9t^2+9}{t^4+9t^2} = 1 + \frac{-9t^2+9}{t^2(t^2+9)}$ (after long division); $\frac{-9t^2+9}{t^2(t^2+9)} = \frac{A}{t} + \frac{B}{t^2} + \frac{Ct+D}{t^2+9} \Rightarrow -9t^2 + 9 = At(t^2+9) + B(t^2+9) + (Ct+D)t^2 = (A+C)t^3 + (B+D)t^2 + 9At + 9B \Rightarrow \left.\begin{array}{rcl} A+C & = & 0 \\ B+D & = & -9 \\ 9A & = & 0 \\ 9B & = & 9 \end{array}\right\} \Rightarrow$

$A = 0 \Rightarrow C = 0;\ B = 1 \Rightarrow D = -10;$ thus, $\frac{t^4+9}{t^4+9t^2} = 1 + \frac{1}{t^2} + \frac{-10}{t^2+9}$

8.3: 14 $\frac{y+4}{y^2+y} = \frac{A}{y} + \frac{B}{y+1} \Rightarrow y+4 = A(y+1) + By;\ y = 0 \Rightarrow A = 4;\ y = -1 \Rightarrow B = \frac{3}{-1} = -3;\ \int_{1/2}^1 \frac{y+4}{y^2+y}dy = 4\int_{1/2}^1 \frac{dy}{y} - 3\int_{1/2}^1] fracdyy + 1 = [4\ln|y| - 3\ln|y+1|]_{1/2}^1 = (4\ln 1 - 3\ln 2) - (4\ln\frac{1}{2} - 3\ln\frac{3}{2}) = \ln\frac{1}{8} - \ln\frac{1}{16} + \ln\frac{27}{8} = \ln(\frac{27}{8}\cdot\frac{1}{8}\cdot 16) = \ln\frac{27}{4}$

8.3: 20 $\frac{x^2}{(x-1)(x^2+2x+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+1} \Rightarrow x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1);\ x = -1 \Rightarrow C = -\frac{1}{2};\ x = 1 \Rightarrow A = \frac{1}{4};$ coefficient for $x^2 = A + B \Rightarrow A + B = 1 \Rightarrow B = \frac{3}{4};\ \int \frac{x^2 dx}{(x-1)(x^2+2x+1)} = \frac{1}{4}\int \frac{dx}{x-1} + \frac{3}{4}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{dx}{(x+1)^2} = \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2(x+1)} + C = \frac{\ln|(x-1)(x+1)^3|}{4} + \frac{1}{2(x+1)} + C$

8.3: 24 $\frac{8x^2+8x+2}{(4x^2+1)^2} = \frac{Ax+B}{4x^2+1} + \frac{Cx+D}{(4x^2+1)^2} \Rightarrow 8x^2 + 8x + 2 = (Ax+B)(4x^2+1) + Cx + D = 4Ax^3 + 4Bx^2 + (A+C)x + (B+D); A = 0,\ B = 2;\ A + C = 8 \Rightarrow C = 8;\ B + D = 2 \Rightarrow D = 0;\ \int \frac{8x^2+8x+2}{(4x^2+1)^2}dx = 2\int \frac{dx}{4x^2+1} + 8\int \frac{x dx}{(4x^2+1)^2} = \tan^{-1} 2x - \frac{1}{4x^2+1} + C$

8.3: 30 $\frac{x^4}{x^2-1} = (x^2+1) + \frac{1}{x^2-1} = (x^2+1) + \frac{1}{(x+1)(x-1)};\ \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \Rightarrow 1 = A(x-1) + B(x+1);\ x = -1 \Rightarrow A = -\frac{1}{2};\ x = 1 \Rightarrow B = \frac{1}{2};\ \int \frac{x^4}{x^2-1}dx = \int(x^2+1)dx - \frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{dx}{x-1} = \frac{1}{3}x^3 + x - \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C = \frac{x^3}{3} + x + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$

8.3: 38 $\int \frac{\sin\theta d\theta}{\cos^2\theta + \cos\theta - 2};\ [\cos\theta = y] \to -\int \frac{dy}{y^2+y-2} = \frac{1}{3}\int \frac{dy}{y+2} - \frac{1}{3}\int \frac{dy}{y-1} = \frac{1}{3}\ln\left|\frac{y+2}{y-1}\right| + C = \frac{1}{3}\ln\left|\frac{\cos\theta+2}{\cos\theta-1}\right| + C$

8.4: 4 $\int_0^{\pi/6} 2\cos^5 3x dx = \int_0^{\pi/6} (\cos^2 3x)^2 \cos 3x \cdot 3dx = \int_0^{\pi/6} (1-\sin^2 3x)^2 \cos 3x \cdot 3dx = \int_0^{\pi/6} (1 - 2\sin^2 3x + \sin^4 3x)\cos 3x \cdot 3dx = \int_0^{\pi/6} \cos 3x \cdot 3dx - 2\int_0^{\pi/6} \sin^2 3x \cos 3x \cdot 3dx + \int_0^{\pi/6} \sin^4 3x \cos 3x \cdot 3dx = \left[\sin 3x - 2\frac{\sin^3 3x}{3} + \frac{\sin^5 3x}{5}\right]_0^{\pi/6} = (1 - \frac{2}{3} + \frac{1}{5}) - (0) = \frac{8}{15}$

8.4: 20 $\int_{-\pi/4}^{\pi/4} \sqrt{\sec^2 x - 1} dx = \int_{-\pi/4}^{\pi/4} |\tan x| dx = -\int_{-\pi/4}^{0} \tan x dx + \int_0^{\pi/4} \tan x dx = [-\ln|\sec x|]_{-\pi/4}^{0} + [-\ln|\sec x|]_0^{\pi/4} = -\ln(1) + \ln\sqrt{2} + \ln\sqrt{2} - \ln(1) = 2\ln\sqrt{2} = \ln 2$

8.4: 30 $\int_{-\pi/4}^{\pi/4} 6\tan^4 x dx = 6\int_{-\pi/4}^{\pi/4} (\sec^2 x - 1)\tan^2 x dx = 6\int_{-\pi/4}^{\pi/4} \sec^2 x \tan^2 x dx - 6\int_{-\pi/4}^{\pi/4} \tan^2 x dx = 6\int_{-\pi/4}^{\pi/4} \sec^2 x \tan^2 x dx - 6\int_{-\pi/4}^{\pi/4} (\sec^2 x - 1) dx = \left[6\frac{\tan^3 x}{3}\right]_{-\pi/4}^{\pi/4} - 6\int_{-\pi/4}^{\pi/4} \sec^2 x dx + 6\int_{-\pi/4}^{\pi/4} dx = 2(1-(-1)) - [6\tan x]_{-\pi/4}^{\pi/4} + [6x]_{-\pi/4}^{\pi/4} = 4 - 6(1-(-1)) + \frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi - 8$

8.4: 36 $\int_0^{\pi/2} \sin x \cos x dx = \frac{1}{2}\int_0^{\pi/2} (\sin 0 + \sin 2x) dx = \frac{1}{2}\int_0^{\pi/2} \sin 2x dx = -\frac{1}{4}[\cos 2x]_0^{\pi/2} = -\frac{1}{4}(-1-1) = \frac{1}{2}$

8.4: 38 $\int_{-\pi/2}^{\pi/2} \cos 7x \cos x dx = \frac{1}{2}\int_{-\pi/2}^{\pi/2} (\cos 6x + \cos 8x) dx = \frac{1}{2}[\frac{1}{6}\sin 6x + \frac{1}{8}\sin 8x]_{-\pi/2}^{\pi/2} = 0$

8.5: 2 $\int \frac{3dy}{\sqrt{1+9y^2}}$; $[3y = x] \to \int \frac{dx}{\sqrt{1+x^2}}$; $x = \tan t,\ -\frac{\pi}{2} < t < \frac{\pi}{2},\ dx = \frac{dt}{\cos^2 t},\ \sqrt{1+x^2} = \frac{1}{\cos t}$; $\int \frac{dx}{\sqrt{1+x^2}} = \int \frac{dt}{\cos^2 t(\frac{1}{\cos t})} = \ln|\sec t + \tan t| + C = \ln\left|\sqrt{x^2+1} + x\right| + C = \ln|\sqrt{1+9y^2} + 3y| + C$

8.5: 8 $t = \frac{1}{3}\sin\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2},\ dt = \frac{1}{3}\cos\theta d\theta,\ \sqrt{1-9t^2} = \cos\theta$; $\int \sqrt{1-9t^2} dt = \frac{1}{3}\int (\cos\theta)(\cos\theta) d\theta = \frac{1}{3}\int \cos^2\theta d\theta = \frac{1}{6}(\theta + \sin\theta\cos\theta) + C = \frac{1}{6}\left[\sin^{-1}(3t) + 3t\sqrt{1-9t^2}\right] + C$

8.5: 16 $x = \tan\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2},\ dx = \sec^2\theta d\theta,\ \sqrt{x^2+1} = \sec\theta$; $\int \frac{dx}{x^2\sqrt{x^2+1}} = \int \frac{\sec^2\theta d\theta}{\tan^2\theta \sec\theta} = \int \frac{\cos\theta d\theta}{\sin^2\theta} = -\frac{1}{\sin\theta} + C = -\frac{\sqrt{x^2+1}}{x} + C$

8.5: 28 $r = \sin\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$; $\int \frac{(1-r^2)^{5/2} dr}{r^8} = \int \frac{\cos^5\theta \cdot \cos\theta d\theta}{\sin^8\theta} = \int \cot^6\theta \csc^2\theta d\theta = -\frac{\cot^7\theta}{7} + C = -\frac{1}{7}\left[\frac{\sqrt{1-r^2}}{r}\right]^7 + C$

8.5: 34 $x = \tan\theta,\ dx = \sec^2\theta d\theta,\ 1 + x^2 = \sec^2\theta$; $\int \frac{dx}{x^2+1} = \int \frac{\sec^2\theta d\theta}{\sec^2\theta} = \theta + C = \tan^{-1} x + C$