

- 8.7: 12 a.  $n = 8 \Rightarrow \Delta x = \frac{3}{8} \Rightarrow \frac{\Delta x}{2} = \frac{3}{16}$ ;  $\Sigma mf(\theta_i) = 1(0) + 2(0.09334) + 2(0.18429) + 2(0.27075) + 2(0.3512) + 2(0.42443) + 2(0.49026) + 2(0.58466) + 1(0.6) = 5.3977 \Rightarrow T = \frac{3}{16}(5.3977) = 1.01207$
- b.  $n = 8 \Rightarrow \Delta x = \frac{3}{8} \Rightarrow \frac{\Delta x}{3} = \frac{1}{8}$ ;  $\Sigma mf(\theta_i) = 1(0) + 4(0.09334) + 2(0.18429) + 4(0.27075) + 2(0.3512) + 4(0.42443) + 2(0.49026) + 4(0.58466) + 1(0.6) = 8.14406 \Rightarrow T = \frac{1}{8}(8.14406) = 1.01801$
- c. Let  $u = 16 + \theta^2 \Rightarrow du = 2\theta d\theta \Rightarrow \frac{1}{2}du = \theta d\theta$ ;  $\theta = 0 \Rightarrow u = 16$ ,  $\theta = 3 \Rightarrow u = 16 + 3^2 = 25$ ;  $\int_0^3 \frac{\theta}{\sqrt{16+\theta^2}} d\theta = \int_{16}^{25} \frac{1}{\sqrt{u}} (\frac{1}{2}du) = \frac{1}{2} \int_{16}^{25} u^{-1/2} du = \left[ \frac{1}{2} \left( \frac{u^{1/2}}{\frac{1}{2}} \right) \right]_{16}^{25} = \sqrt{25} - \sqrt{16} = 1$ ;  $E_T = \int_0^3 \frac{\theta}{\sqrt{16+\theta^2}} d\theta - T \approx 1 - 1.01207 = -0.01207$ ;  $E_S = \int_0^3 \frac{\theta}{\sqrt{16+\theta^2}} d\theta - S \approx 1 - 1.01801 = -0.01801$
- 8.7: 20 a.  $M = 6$ (see Exercise 6); Then  $\Delta x = \frac{2}{n} \Rightarrow |E_T| \leq \frac{2}{12} \left(\frac{2}{n}\right)^2 (6) = \frac{4}{n^2} < 10^{-4} \Rightarrow n^2 > 4(10^4) \Rightarrow n > \sqrt{4(10^4)} = 200$ , so let  $n = 201$
- b.  $M = 0$ (see Exercise 6); Then  $n = 2$  ( $n$  must be even)  $\Rightarrow \Delta x = 1 \Rightarrow |E_s| = \frac{2}{180}(1)^4(0) = 0 < 10^{-4}$
- 8.7: 26 a.  $f(x) = \cos(x + \pi) \Rightarrow f'(x) = -\sin(x + \pi) \Rightarrow f''(x) = -\cos(x + \pi) \Rightarrow M = 1$ . Then  $\Delta x = \frac{2}{n} \Rightarrow |E_T| \leq \frac{2}{12} \left(\frac{2}{n}\right)^2 (1) = \frac{8}{12n^2} < 10^{-4} \Rightarrow n^2 > \frac{8(10^4)}{12} \Rightarrow n > \sqrt{\frac{8(10^4)}{12}} \Rightarrow n > 81.6$ , so let  $n = 82$ .
- b.  $f^{(3)}(x) = \sin(x + \pi) \Rightarrow f^{(4)}(x) = \cos(x + \pi) \Rightarrow M = 1$ . Then  $\Delta x = \frac{2}{n} \Rightarrow |E_s| \leq \frac{2}{180} \left(\frac{2}{n}\right)^4 (1) = \frac{32}{180n^4} < 10^{-4} \Rightarrow n^4 > \frac{32(10^4)}{180} \Rightarrow n > \sqrt[4]{\frac{32(10^4)}{180}} \Rightarrow n > 6.49$ , so let  $n = 8$  ( $n$  must be even)
- 8.7: 28a Using Trapezoid Rule,  $\Delta x = 200 \Rightarrow \frac{\Delta x}{2} = \frac{200}{2} = 100$ ;  $\Sigma mf(x_i) = 13,180 \Rightarrow \text{Area} \approx 100(13,180) = 1,318,000\text{ft}^2$ . Since the average depth = 20ft, we obtain Volume  $\approx 20(\text{Area}) \approx 26,360,000\text{ft}^3$ .

	$x_i$	$f(x_i)$	$m$	$mf(x_i)$
$x_0$	0	0	1	0
$x_1$	200	520	2	1040
$x_2$	400	800	2	1600
$x_3$	600	1000	2	2000
$x_4$	800	1140	2	2280
$x_5$	1000	1160	2	2320
$x_6$	1200	1110	2	2220
$x_7$	1400	860	2	1720
$x_8$	1600	0	2	0

- 8.7: 32  $\frac{24}{2}[0.019 + 2(0.020) + 2(0.021) + \dots + 2(0.031) + 0.035] = 4.2 \text{ L}$
- 8.8: 6  $\int_{-8}^1 \frac{dx}{x^{1/3}} = \int_{-8}^0 \frac{dx}{x^{1/3}} + \int_0^1 \frac{dx}{x^{1/3}} = \lim_{b \rightarrow 0^-} \left[ \frac{3}{2}x^{2/3} \right]_{-8}^b + \lim_{c \rightarrow 0^+} \left[ \frac{3}{2}x^{2/3} \right]_c^1 = \lim_{b \rightarrow 0^-} \left[ \frac{3}{2}b^{2/3} - \frac{3}{2}(-8)^{2/3} \right] + \lim_{c \rightarrow 0^+} \left[ \frac{3}{2}(1)^{2/3} - \frac{3}{2}c^{2/3} \right] = \left[ 0 - \frac{3}{2}(4) \right] + \left( \frac{3}{2} - 0 \right) = -\frac{9}{2}$
- 8.8: 24  $\int_{-\infty}^{\infty} 2xe^{-x^2} dx = \int_{-\infty}^0 2xe^{-x^2} dx + \int_0^{\infty} 2xe^{-x^2} dx = \lim_{b \rightarrow -\infty} \left[ -e^{-x^2} \right]_b^0 + \lim_{c \rightarrow \infty} \left[ -e^{-x^2} \right]_0^c = \lim_{b \rightarrow -\infty} \left[ -1 - (-e^{-b^2}) \right] + \lim_{c \rightarrow \infty} \left[ -e^{-c^2} - (-1) \right] = (-1 - 0) + (0 + 1) = 0$
- 8.8: 26  $\int_0^1 (-\ln x) dx = \lim_{b \rightarrow 0^+} [x - x \ln x]_b^1 = [1 - 1 \ln 1] - \lim_{b \rightarrow 0^+} [b - b \ln b] = 1 - 0 + \lim_{b \rightarrow 0^+} \frac{\ln b}{\frac{1}{b}} = 1 + \lim_{b \rightarrow 0^+} \frac{\frac{1}{b}}{-\frac{1}{b^2}} = 1 - \lim_{b \rightarrow 0^+} b = 1 - 0 = 1$
- 8.8: 30  $\int_2^4 \frac{dt}{t\sqrt{t^2-4}} = \lim_{b \rightarrow 2^+} \left[ \frac{1}{2} \sec^{-1} \frac{t}{2} \right]_b^4 = \lim_{b \rightarrow 2^+} \left[ \left( \frac{1}{2} \sec^{-1} \frac{4}{2} \right) - \frac{1}{2} \sec^{-1} \left( \frac{b}{2} \right) \right] = \frac{1}{2} \left( \frac{\pi}{3} \right) - \frac{1}{2} \cdot 0 = \frac{\pi}{6}$

8.8: 52  $\int_2^\infty \frac{dx}{\sqrt{x^2-1}}$ ;  $\lim_{x \rightarrow \infty} \frac{(\frac{1}{\sqrt{x^2-1}})}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2-1}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1-\frac{1}{x^2}}} = 1$ ;  $\int_2^\infty \frac{1}{x} dx = \lim_{b \rightarrow \infty} [\ln b]_2^b = \infty$ ,  
which diverges  $\Rightarrow \int_2^\infty \frac{dx}{\sqrt{x^2-1}}$  diverges by Limit Comparison Test.

8.8: 54  $\int_2^\infty \frac{x dx}{\sqrt{x^4-1}}$ ;  $\lim_{x \rightarrow \infty} \frac{\frac{x}{\sqrt{x^4-1}}}{\frac{x}{\sqrt{x^4}}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^4}}{\sqrt{x^4-1}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1-\frac{1}{x^4}}} = 1$ ;  $\int_2^\infty \frac{x dx}{\sqrt{x^4}} = \int_2^\infty \frac{dx}{x} = \lim_{b \rightarrow \infty} [\ln x]_2^b = \infty$ , which diverges  $\Rightarrow \int_2^\infty \frac{x dx}{\sqrt{x^4-1}}$  diverges by Limit Comparison Test.

8.8: 58  $\int_2^\infty \frac{dx}{\ln x}$ ;  $0 < \frac{1}{x} < \frac{1}{\ln x}$  for  $x > 2$  and  $\int_2^\infty \frac{dx}{x}$  diverges  $\Rightarrow \int_2^\infty \frac{dx}{\ln x}$  diverges by Direct Comparison Test.

8.8: 64  $\int_{-\infty}^\infty \frac{dx}{e^x+e^{-x}} = 2 \int_0^\infty \frac{dx}{e^x+e^{-x}}$ ;  $0 < \frac{1}{e^x+e^{-x}} < \frac{1}{e^x}$  for  $x > 0$ ;  $\int_0^\infty \frac{dx}{e^x}$  converges  $\Rightarrow 2 \int_0^\infty \frac{dx}{e^x+e^{-x}}$  converges by Direct Comparison Test.