

11.1: 16  $a_n = \frac{(-1)^{n+1}}{n^2}$ ,  $n = 1, 2, \dots$

11.1: 32  $\lim_{n \rightarrow \infty} (-1)^n (1 - \frac{1}{n})$  does not exist  $\Rightarrow$  diverges.

11.1: 50  $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = \lim_{n \rightarrow \infty} \left[1 + \frac{(-1)}{n}\right]^n = e^{-1} \Rightarrow$  converges. (Theorem 5, #5)

11.1: 82  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-1} - \sqrt{n^2+n}} = \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2-1} - \sqrt{n^2+n}}\right) \left(\frac{\sqrt{n^2-1} + \sqrt{n^2+n}}{\sqrt{n^2-1} + \sqrt{n^2+n}}\right) = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-1} + \sqrt{n^2+n}}{1-n} =$   
 $\lim_{n \rightarrow \infty} \frac{\sqrt{1 - \frac{1}{n^2}} + \sqrt{1 + \frac{1}{n}}}{-\frac{1}{n} - 1} = -2 \Rightarrow$  converges.

11.2: 6  $\frac{5}{n(n+1)} = \frac{5}{n} - \frac{5}{n+1} \Rightarrow S_n = (5 - \frac{5}{2}) + (\frac{5}{2} - \frac{5}{3}) + (\frac{5}{3} - \frac{5}{4}) + \dots + (\frac{5}{n-1} - \frac{5}{n}) + (\frac{5}{n} - \frac{5}{n+1}) = 5 - \frac{5}{n+1} \Rightarrow \lim_{n \rightarrow \infty} S_n = 5$

11.2: 12  $(5-1) + (\frac{5}{2} - \frac{1}{3}) + (\frac{5}{4} - \frac{1}{9}) + (\frac{5}{8} - \frac{1}{27}) + \dots$ , is the difference of two geometric series; the sum is  $\frac{5}{1 - (\frac{1}{2})} - \frac{1}{1 - (\frac{1}{3})} = 10 - \frac{3}{2} = \frac{17}{2}$

11.2: 14  $2 + \frac{4}{5} + \frac{8}{25} + \frac{16}{125} + \dots = 2(1 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \dots)$ ; the sum of this geometric series is  $2 \left(\frac{1}{1 - (\frac{2}{5})}\right) = \frac{10}{3}$

11.2: 16  $\frac{6}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1} = \frac{A(2n+1) + B(2n-1)}{(2n-1)(2n+1)} \Rightarrow A(2n+1) + B(2n-1) = 6 \Rightarrow (2A+2B)n + (A-B) = 6 \Rightarrow \begin{cases} 2A+2B = 0 \\ A-B = 6 \end{cases} \Rightarrow \begin{cases} A+B = 0 \\ A-B = 6 \end{cases} \Rightarrow 2A = 6 \Rightarrow A = 3 \text{ and } B = -3. \text{ Hence,}$

$$\sum_{n=1}^k \frac{6}{(2n-1)(2n+1)} = 3 \sum_{n=1}^k \frac{1}{2n-1} + \frac{1}{2n+1} = 3 \left( \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \dots - \frac{1}{2(k-1)+1} + \frac{1}{2k-1} - \frac{1}{2k+1} \right) =$$

$$3 \left( 1 - \frac{1}{2k+1} \right) \Rightarrow \text{the sum is } \lim_{k \rightarrow \infty} 3 \left( 1 - \frac{1}{2k+1} \right) = 3.$$

11.2: 30  $\lim_{n \rightarrow \infty} \ln \frac{1}{n} = -\infty \neq 0 \Rightarrow$  diverges.

11.2: 34  $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{-1}{n}\right)^n = e^{-1} \neq 0 \Rightarrow$  diverges.

11.2: 48  $a = 1$ ,  $r = \frac{3-x}{2}$ ; converges to  $\frac{1}{1 - (\frac{3-x}{2})} = \frac{2}{x-1}$  for  $|\frac{3-x}{2}| < 1$  or  $1 < x < 5$ .