11.3: 2 converges; a geometric series with $r = \frac{1}{e} < 1$

11.3: 6 converges; $\sum_{n=1}^{\infty} \frac{-2}{n\sqrt{n}} = -2\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$, which is a convergent p-series ($p = \frac{3}{2}$).

11.3: 18 diverges; $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e \neq 0$

11.3: 28 diverges by the Integral Test; $\int_1^{\infty} \frac{x}{x^2+1}dx$; $\begin{bmatrix} u & = & x^2+1 \\ du & = & 2xdx \end{bmatrix} \rightarrow \frac{1}{2}\int_2^{\infty} \frac{du}{4} = \lim_{b\to\infty} \left[\frac{1}{2}\ln u\right]_2^b = \lim_{b\to\infty} \frac{1}{2}(\ln b - \ln 2) = \infty$

11.3: 39

a. $\int_2^{\infty} \frac{dx}{x(\ln x)^p}$; $\begin{bmatrix} u & = & \ln x \\ du & = & \frac{dx}{x} \end{bmatrix} \rightarrow \int_{\ln 2}^{\infty} u^{-p}du = \lim_{b\to\infty} \left[\frac{u^{-p+1}}{-p+1}\right]_{\ln 2}^b = \lim_{b\to\infty} (\frac{1}{1-p})[b^{-p+1} - (\ln 2)^{-p+1}]$ $= \begin{cases} \frac{1}{p-1}(\ln 2)^{-p+1} & , p > 1 \\ \infty & , p < 1 \end{cases}$ $\Rightarrow$ the improper integral converges if $p > 1$ and diverges if $p < 1$.

For $p = 1$; $\int_2^{\infty} \frac{dx}{x\ln x} = \lim_{b\to\infty} [\ln(\ln x)]_2^b = \lim_{b\to\infty} [\ln(\ln b) - \ln(\ln 2)] = \infty$, so the improper integral diverges if $p = 1$.

b. Since the series and the integral converges or diverges together, $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges if and only if $p > 1$.

11.4: 4 converges by the Direct Comparison Test; $\frac{1+\cos n}{n^2} \leq \frac{2}{n^2}$ and the p-series $\Sigma \frac{1}{n^2}$ converges.

11.4: 9 diverges by the Direct Comparison Test; $n > \ln n \Rightarrow \ln n > \ln\ln n \Rightarrow \frac{1}{n} < \frac{1}{\ln n} < \frac{1}{\ln(\ln n)}$ and $\sum_{n=3}^{\infty} \frac{1}{n}$ diverges.

11.4: 16 diverges by the Limit Comparison Test (part 3) with $\frac{1}{n}$, the nth term of the divergent harmonic series; $\lim_{n\to\infty} \frac{\frac{1}{1+\ln n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{(1+\ln n)^2} = \lim_{n\to\infty} \frac{1}{\frac{2(1+\ln n)}{n}} = \lim_{n\to\infty} \frac{n}{2(1+\ln n)} = \lim_{n\to\infty} \frac{2}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{2} = \infty$

11.4: 35 $\frac{1}{1+2+3+\ldots+n} = \frac{1}{\frac{n(n+1)}{2}} = \frac{2}{n(n+1)}$. The series converges by the Limit Comparison Test (part 1) with $\frac{1}{n^2}$: $\lim_{n\to\infty} \frac{\frac{2}{n(n+1)}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{2n^2}{n^2+n} = \lim_{n\to\infty} \frac{4n}{2n+1} = \lim_{n\to\infty} \frac{4}{2} = 2.$

11.4: 36 $\frac{1}{1^2+2^2+3^2+\ldots+n^2} = \frac{1}{\frac{n(n+1)(2n+1)}{6}} = \frac{6}{n(n+1)(2n+1)} \leq \frac{6}{n^3} \Rightarrow$ the series converges by the Direct Comparison Test.

11.5: 10 diverges by the nth Term Test; $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 - \frac{1}{3n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{-\frac{1}{3}}{n}\right)^n = e^{-1/3} \approx 0.72 \neq 0$

11.5: 20 converges by the Ratio Test; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)2^{n+1}(n+2)!}{3^{n+1}(n+1)!} \cdot \frac{3^n n!}{n2^n(n+1)!} = \lim_{n\to\infty} (\frac{n+1}{n})(\frac{2}{3})(\frac{n+2}{n+1}) = \frac{2}{3} < 1$

11.5: 22 converges by the Ratio Test; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \frac{1}{\left(\frac{n}{n+1}\right)^n} =$

$\lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{1}{e} < 1$

11.5: 23 converges by the Root Test; $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{(\ln n)^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{\ln n} = \lim_{n\to\infty} \frac{1}{\ln n} = 0 < 1$